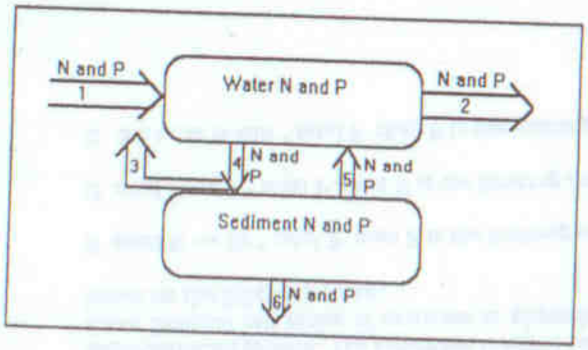
Model description

Lake Model consists of a combination of two kinds of models: a causal dynamical model, and a set of associated empirical models. The dynamical model integrates the pools nitrogen and phosphorus in water and sediment in time as functions of the massflows. The empirical models are simple regressions made from data of simple physical and chemical characteristics of a number of lakes.

The dynamical model is a modification of the general model made by Vollenweider (1975). While Vollenweiders model was only concerned with phosphorus, which is the limiting nutrient in most freshwater bodies, *Lake Model* has included nitrogen as well. The conceptual diagram for *Lake Model* is shown below.



The nitrogen and the phosphorus submodels are almost identical. The only difference is the denitrification process included in the nitrogen submodel.

The mathematical formulation of the nitrogen submodel, referring to the numbers in the diagram:

1) Nload/z	(loading, mg/l/year)
2) (1/Wres) * Nwat * a	(hydraulic outwash, mg/l/year)
3) Denit/z	(denitrification, mg/l/year)
4) NWat * SedRate /z	(sedimentation, mg/l/year)
5) NSed * NRel/z	(sediment release, mg/l/year)
6) (1-NBound) * Nsed	(demobilization from sediment, g/m^2/year)

These processes form the two differential equations :

$$\frac{dNwat}{dt} = ((Nload-Denit)+Nrel*NSed)/z-(1/Wres)*Nwat*a-(1/z)*SedRate*Nwat$$

$$\frac{dNSed}{dt} = SedRate*Nwat*(1-NBound)-Nrel*Nsed$$

where

NWat is total nitrogen (mg/l) in the water column.
Nsed is nitrogen (g/m^2) in sediment.
Nload is the nitrogen input (g/m^2/year) to the lake
Nrel is the sediment release rate (/year) of nitrogen.
Nbound is the ratio of immobilized sedimentated nitrogen.
z is mean depth (m) of the lake.
Wres is the mean residence time (year) of the water.
SedRate is the mean sedimentation rate. (m/year)
a is a correction factor of nutrient output due to thermocline formation.

Note that the units for the sediment pools are g/m^2 and mg/l for the water column pools. The denitrification is described by the empirical model:

$$Denit = NLoad \cdot 0.34 * Wres^{-0.16} * z^{0.17}$$

Except for the denitrification, the phosphorus submodel is formulated analogous to the nitrogen submodel. The equations are

$$\frac{dPWat}{dt} = (Pload+Prel * Psed)/z-1/Wres * Pwat * a-1/z * SedRate * Pwat$$

$$\frac{dPSed}{dt} = SedRate * Pwat * (1-PBound) - Prel * Psed$$

where

PWat is total nitrogen (mg/l) in the water column.
Psed is nitrogen (g/m^2) in sediment.
Pload is the nitrogen input (g/m^2/year) to the lake.
Prel is the sediment release rate (/year) of nitrogen.
Pbound is the immobilized ratio of sedimentated nitrogen.

The empirical models are a number af relations made from statistical regression analyses (Edmondson, 1986).

Chlorophyll (mg/l)	$= 0.000073 * (TP * 1000)^{1.4}$
Zooplankton (mg/l)	$= 0.038 * (TP * 1000)^{0.64}$
Fish (mg ww/m^2)	$= 0.810 * (TP * 1000)^{0.71}$
Average primary production (mg/l/day)	$= (10000 * TP - 79)/1000$
Maximum primary production (mg/l/day)	$= (20000 * TP - 77)/1000$
Average fish yield (mg ww/m^2/year)	$= 7.1 * TP$

TP is the total phosphorus. *Lake Model* is supplied with an algorithm to decide if phosphorus and/or nitrogen are limiting nutrients to the phytoplankton growth. The algorithm is based on the knowledge about the mean internal cell ratios of nutrients in phytoplankton. The algorithm is based on the following rules:

If total N >= 10 * total P then P is the limiting nutrient

If total N <= 5 * total P then N is the limiting nutrient

If 5 < total N <10 * total P then P is the limiting nutrient

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