BACKWATER CURVES

Gradually-varied flows

Differential equation:

$$\frac{dH}{dx} = -S_f \qquad (1)$$

where: $$H = \frac{U^2}{2g} + z + y \qquad (2)$$

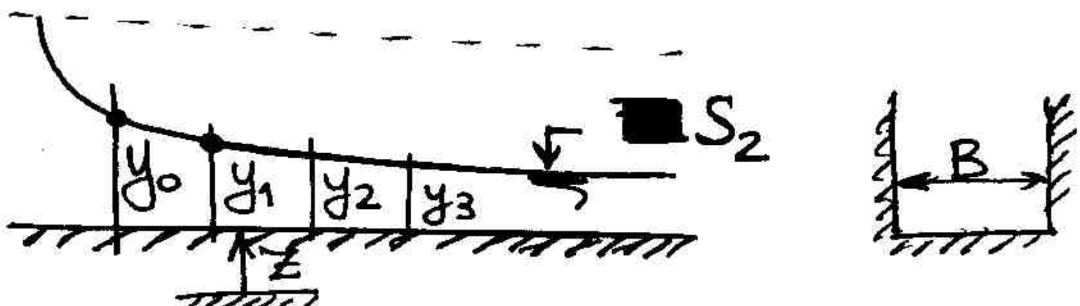
and: $$S_f = C_f \frac{U^2}{gy} \qquad (3)$$

We can discretize (1) using <u>any</u> of the methods we saw for $\frac{du}{dt} = -Ku$. Notice that the two equations look very similar. We can use a forward, backward, or a centered scheme. Using a centered scheme:

$$\frac{H_{j+1} - H_j}{\Delta x} = -\frac{1}{2}\left(S_{f_j} + S_{f_{j+1}}\right) \qquad (4)$$

H is a function of y. S_f is also a function of y. Recall that in backwater curves our objective is to compute the variation of y with distance. The computation starts at one of the [redacted] boundaries of the cur ve and proceeds to the other boundary.

Example:



The problem with (4) is that, since $H = H(y)$ and $S_f(y)$, y is at both sides of the equal sign.

<u>First method</u>: We can circumvent this problem by <u>fixing y</u>. Say $y_0 = 10$ m; let's fix $y_1 = 9.9$ m, $y_2 = 9.8$ m, etc. If we know y, and the channel has a rectangular cross section, we can compute the area as: $A_0 = By_0$, $A_1 = By_1$, $A_2 = By_2$, etc. Further, if the discharge per unit width, q_w, is known, we can compute: $U_0 = \dfrac{q_w}{y_0}$, $U_1 = \dfrac{q_w}{y_1}$, $U_2 = \dfrac{q_w}{y_2}$, etc.

If C_f is given, we can compute H_0, H_1, H_2, etc., and S_{f0}, S_{f1}, S_{f2}, etc.

For the first two cross sections:

$$\frac{H_1 - H_0}{\Delta x} = -\frac{1}{2}\left(S_{f_0} + S_{f_1}\right) \quad (5)$$

From (5), we know everything except Δx. So we can compute Δx from (5). This means that we compute the distance at which $y_1 = 9.9$ m occurs from $y_0 = 10$ m.

This method is called explicit, because we can compute Δx from known values:

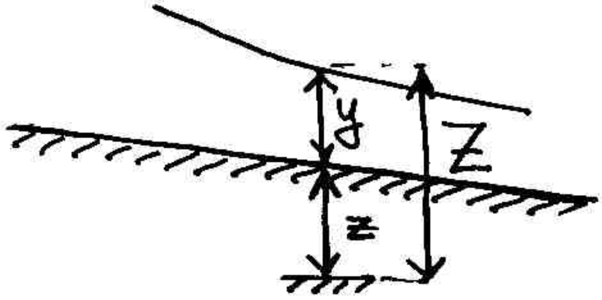
$$\Delta x = \frac{(H_0 - H_1)\,2}{(S_{f_0} + S_{f_1})} \quad (6)$$

We repeat the procedure between H_1 and H_2, H_2 and H_3, etc. In this way, we build the curve.

Second method: Equation (4) is also used. Let's denote Z the elevation of the water surface from the datum:

$$Z = z + y$$

$$H = Z + \frac{U^2}{2g} \quad (7)$$



From (4):

$$H_{j+1} = H_j - \frac{1}{2}\left(S_{f_j} + S_{f_{j+1}}\right)\Delta x \quad (8)$$

This method is best suited for natural channels. This method is based on trial and error procedure (iterative). Thus, it is called implicit method. Procedure:

1) We guess a value of Z[redacted]$_1$ (recall eqn. (5)) Since we know z_1, we can obtain y_1. Having y_1, we compute A_1, U_1, S_{f_1} and H_1^1. All these values at 0 are known because it is the initial condition. But this value of Z_1 corresponds to a given Δx that we specify. Thus, all these values give us another value of H_1^2. If H_1^2 is different from H_1^1, we need to vary our guess of [redacted] Z_1. The iteration continues until

$$\left|\frac{H_1^{m+1} - H_1^{m}}{H_1^{m+1}}\right| < \text{tol.}$$

Then, we need to move to another pair of cross sections and repeat the procedure.

Examples: Taken from Open-channel hydraulics, by Ven Te Chow, McGraw-Hill.

$$S_f = C_f \frac{U^2}{g y} = \frac{m^2 U^2}{\text{[redacted]}\, R^{4/3}}$$

Example of explicit method

⑥

TABLE 10-4. COMPUTATION OF THE FLOW PROFILE BY THE DIRECT STEP METHOD FOR EXAMPLE 10-7
$Q = 400$ cfs $n = 0.025$ $S_0 = 0.0016$ $\alpha = 1.10$ $y_c = 2.22$ ft $y_n = 3.36$ ft

y (1)	A (2)	R (3)	$R^{4/3}$ (4)	V (5)	$\alpha V^2/2g$ (6)	E (7)	ΔE (8)	S_f (9)	$\bar{S}_f$ (10)	$S_0 - \bar{S}_f$ (11)	Δx (12)	x (13)
5.00	150.00	3.54	5.40	2.667	0.1217	5.1217	......	0.000370				
4.80	142.08	3.43	5.17	2.819	0.1356	4.9356	0.1861	0.000433	0.000402	0.001198	155	155
4.60	134.32	3.31	4.94	2.979	0.1517	4.7517	0.1839	0.000507	0.000470	0.001130	163	318
4.40	126.72	3.19	4.70	3.156	0.1706	4.5706	0.1811	0.000598	0.000553	0.001047	173	491
4.20	119.28	3.08	4.50	3.354	0.1925	4.3925	0.1781	0.000705	0.000652	0.000948	188	679
4.00	112.00	2.96	4.25	3.572	0.2184	4.2184	0.1741	0.000850	0.000778	0.000822	212	891
3.80	104.88	2.84	4.02	3.814	0.2490	4.0490	0.1694	0.001020	0.000935	0.000665	255	1,146
3.70	101.38	2.77	3.88	3.948	0.2664	3.9664	0.0826	0.001132	0.001076	0.000524	158	1,304
3.60	97.92	2.71	3.78	4.085	0.2856	3.8856	0.0808	0.001244	0.001188	0.000412	196	1,500
3.55	96.21	2.68	3.72	4.158	0.2958	3.8458	0.0398	0.001310	0.001277	0.000323	123	1,623
3.50	94.50	2.65	3.66	4.233	0.3067	3.8067	0.0391	0.001382	0.001346	0.000254	154	1,777
3.47	93.48	2.63	3.63	4.278	0.3131	3.7831	0.0236	0.001427	0.001405	0.000195	121	1,898
3.44	92.45	2.61	3.59	4.326	0.3202	3.7602	0.0229	0.001471	0.001449	0.000151	152	2,050
3.42	91.80	2.60	3.57	4.357	0.3246	3.7446	0.0156	0.001500	0.001486	0.000114	137	2,187
3.40	91.12	2.59	3.55	4.388	0.3292	3.7292	0.0154	0.001535	0.001518	0.000082	188	2,375

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Another example of explicit method (7)

TABLE 10-5. COMPUTATION OF THE FLOW PROFILE FOR EXAMPLE 10-8 BY THE DIRECT STEP METHOD

$Q = 252$ cfs $\quad n = 0.012 \quad S_0 = 0.02 \quad \alpha = 1.0 \quad y_c = 4.35$ ft $\quad y_n = 2.60$ ft

y/D	y	A	R	$R^{4/3}$	V	$\alpha V^2/2g$	E	ΔE	S_f	$\bar{S}_f$	$S_0 - \bar{S}_f$	Δx	x
0.725	4.35	21.95	1.794	2.180	11.48	2.048	6.398		0.00392				
0.70	4.20	21.13	1.777	2.154	11.93	2.211	6.411	0.013	0.00429	0.00411	0.01589	0.8	0.8
0.65	3.90	19.45	1.728	2.073	12.96	2.609	6.509	0.098	0.00525	0.00477	0.01523	6.4	7.2
0.60	3.60	17.71	1.666	1.976	14.23	3.145	6.745	0.236	0.00666	0.00596	0.01404	16.8	24.0
0.55	3.30	15.93	1.590	1.855	15.85	3.901	7.201	0.456	0.00880	0.00773	0.01227	37.2	61.2
0.50	3.00	14.13	1.500	1.717	17.85	4.947	7.947	0.746	0.01202	0.01041	0.00959	77.8	139.0
0.48	2.88	13.42	1.460	1.656	18.76	5.465	8.345	0.398	0.01378	0.01290	0.00710	56.1	195.1
0.47	2.82	13.06	1.440	1.626	19.30	5.785	8.605	0.260	0.01486	0.01432	0.00568	45.8	240.9
0.46	2.76	12.70	1.420	1.596	19.85	6.119	8.879	0.274	0.01600	0.01543	0.00457	60.0	300.9

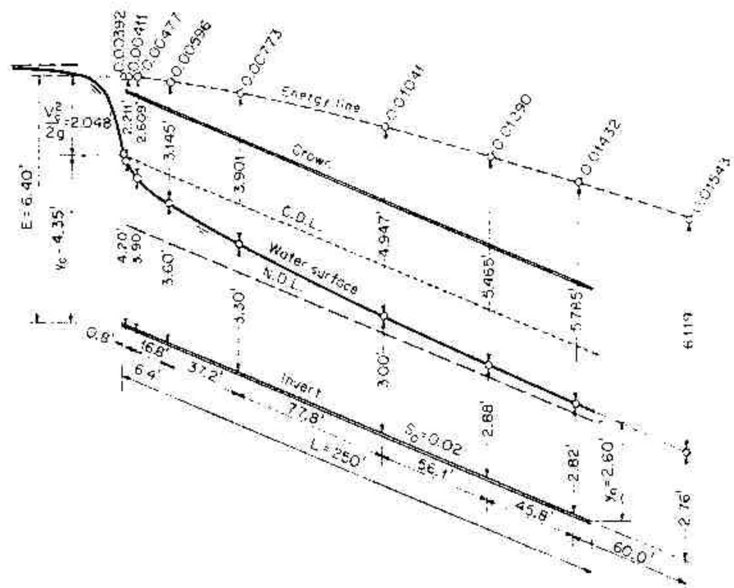

FIG. 10-7. An $S2$ flow profile computed by the direct step method.

Solution. The step computations are arranged in tabular form, as shown in Table 10-6. Values in each column of the table are explained as follows:

Col. 1. Section identified by station number such as "station 1 + 55." The location of the stations is fixed at the distances determined in Example 10-7 in order to compare the procedure with that of the direct step method.

Col. 2. Water-surface elevation at the station. A trial value is first entered in this column; this will be verified or rejected on the basis of the computations made in the remaining columns of the table. For the first step, this elevation must be given or assumed. Since the elevation of the dam site is 600 m.s.l. and the height of the dam is 5 ft, the first entry is 605.00 m.s.l. When the trial value in the second step has been verified, it becomes the basis for the verification of the trial value in the next step, and so on.

Col. 3. Depth of flow in ft, corresponding to the water-surface elevation in col. 2. For instance, the depth of flow at station 1 + 55 is equal to water-surface elevation minus elevation at the dam site minus (distance from the dam site times bed slope), or $605.048 - 600.000 - 155 \times 0.0016 = 4.80$ ft.

Col. 4. Water area corresponding to y in col. 3

Col. 5. Mean velocity equal to the given discharge 400 cfs divided by the water area in col. 4

Col. 6. Velocity head in ft, corresponding to the velocity in col. 5

Col. 7. Total head computed by Eq. (10-47), equal to the sum of Z in col. 2 and the velocity head in col. 6

Col. 8. Hydraulic radius in ft, corresponding to y in col. 3

Col. 9. Four-thirds power of the hydraulic radius

Col. 10. Friction slope computed by Eq. (9-8), with $n = 0.025$, V from col. 5, and $R^{4/3}$ from col. 9

Col. 11. Average friction slope through the reach between the sections in each step, approximately equal to the arithmetic mean of the friction slope just computed in col. 10 and that of the previous step

Col. 12. Length of the reach between the sections, equal to the difference in station numbers between the stations

Col. 13. Friction loss in the reach, equal to the product of the values in cols. 11 and 12.

Col. 14. Eddy loss in the reach, equal to zero

Col. 15. Elevation of the total head in ft. This is computed by Eq. (10-49), that is, by adding the values of h_f and h_e in cols. 13 and 14 to the elevation at the lower end of the reach, which is found in col. 15 of the previous reach. If the value so obtained does not agree closely with that entered in col. 7, a new trial value of the water-surface elevation is assumed, and so on, until agreement is obtained. The value that leads to agreement is the correct water-surface elevation. The computation may then proceed to the next step. The computed flow profile is practically identical with that obtained by the graphical-integration method shown in Fig. 10-3.

10-5. Computation of a Family of Flow Profiles. In previous articles methods were described for determining a single flow profile. Frequently, several flow profiles, or a family of flow profiles, are desired for various conditions of stage and discharge. An example of this type of problem is the determination of the economical height of a dam, where the initial elevation is indeterminate and, hence, a number of flow profiles may have to be computed for the same discharge with different assumed

9

Implicit

TABLE 10-6. COMPUTATION OF THE FLOW PROFILE FOR EXAMPLE 10-9 BY THE $\boxed{\text{STANDARD STEP}}$ METHOD

$Q = 400$ cfs $n = 0.025$ $S_0 = 0.0016$ $\alpha = 1.10$ $h_e = 0$ $y_c = 2.22$ ft $y_n = 3.36$ ft

Station	Z	y	A	V	$\alpha V^2/2g$	H	R	$R^{4/3}$	S_f	$\bar{S}_f$	Δx	h_f	h_e	H
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)
0 + 00	605.000	5.00	150.00	2.667	0.1217	605.122	3.54	5.40	0.000370	....	...	.....	...	605.122
1 + 55	605.048	4.80	142.08	2.819	0.1356	605.184	3.43	5.17	0.000433	0.000402	155	0.062	0	605.184
3 + 18	605.109	4.60	134.32	2.979	0.1517	605.261	3.31	4.92	0.000507	0.000470	163	0.077	0	605.261
4 + 91	605.186	4.40	126.72	3.156	0.1706	605.357	3.19	4.70	0.000598	0.000553	173	0.096	0	605.357
6 + 79	605.286	4.20	119.28	3.354	0.1925	605.479	3.08	4.50	0.000705	0.000652	188	0.122	0	605.479
8 + 91	605.426	4.00	112.00	3.572	0.2184	605.644	2.96	4.25	0.000850	0.000778	212	0.165	0	605.644
11 + 46	605.633	3.80	104.88	3.814	0.2490	605.882	2.84	4.02	0.001020	0.000935	255	0.238	0	605.882
13 + 04	605.786	3.70	101.38	3.948	0.2664	606.052	2.77	3.88	0.001132	0.001076	158	0.170	0	606.052
15 + 00	605.999	3.60	97.92	4.085	0.2856	606.285	2.71	3.78	0.001244	0.001188	196	0.233	0	606.285
16 + 23	606.146	3.55	96.21	4.158	0.2958	606.442	2.68	3.72	0.001310	0.001277	123	0.157	0	606.442
17 + 77	606.343	3.50	94.50	4.233	0.3067	606.650	2.65	3.66	0.001382	0.001346	154	0.208	0	606.650
18 + 98	606.507	3.47	93.48	4.278	0.3131	606.820	2.63	3.63	0.001427	0.001405	121	0.170	0	606.820
20 + 50	606.720	3.44	92.45	4.326	0.3202	607.040	2.61	3.59	0.001471	0.001449	152	0.220	0	607.040
21 + 87	606.919	3.42	91.80	4.357	0.3246	607.244	2.60	3.57	0.001500	0.001486	137	0.204	0	607.244
23 + 75	607.201	3.40	91.12	4.388	0.3292	607.530	2.59	3.55	0.001535	0.001518	188	0.286	0	607.530