QUESTIONS FOR THE TESTS-ECI 146

IMPORTANT: Please note that the questions in the exams WILL BE DIFFERENT than these.

(These questions are valid for the midterm *and* for the final exam)
Remember: A) both tests will be closed books-closed notes.
B) If you do *not* recall a formula, *you can ask me during the test*.
C) I can combine any of these questions, alter them or formulate similar ones.

1. Develop a flow chart for the method of **bisection** of *a general* implicit algebraic equation, considering all possible cases that may take place. Explain, after the chart, the rationale behind those cases.

2. Develop a flow chart for the method of **Regula Falsi** to solve the *Colebrook-White* equation. Devise a way of telling the program to consider the case of a laminar flow (we have seen that the Colebrook-White formula works very well for *turbulent* flow). For a laminar flow, the program should simply compute: $f = 64/\mathrm{Re}$.

3. Develop a flow chart for the method of **Regula Falsi** to solve a *general* algebraic implicit equation, considering all possible cases that may take place. Explain, after the chart, the rationale behind those cases.

4. Explain with two graphs, what the difference in operation between the **Iteration-of-a-point (fixed-point)** method and the **Newton-Raphson** method is. Use words to explain the graphs and how the methods are constructed. Why does the Newton-Raphson method "go through the slope"? (Please explain how the Newton-Raphson equation is derived from geometrical considerations.)

5. Develop a flow chart for a code in which you apply any beginning method to *approximate* the root of a non-linear equation, and then you use the **Fixed-point** method to *refine* the root value. To accomplish that, limit the number of iterations with the beginning method to a "reasonable" amount, and then begin with the **Fixed-point** method. Judge which number of iterations would be a "reasonable" amount.

6. Develop a flow chart for the method of **Newton-Raphson** to solve a *general* algebraic implicit equation.

7. Define simulation. Which types of simulation do you know? Explain the advantages and disadvantages of the numerical simulation with respect to other alternatives. Give examples of those techniques.

8. Which of the methods for solving implicit, algebraic equations we saw in class is more accurate? Which one is more robust? Explain. What do we understand by "robustness"?

9. Explain the meaning of the main mechanisms that characterize the transport of a pollutant in a water body. You can use words and drawings. Determine which of these mechanisms would be more important in: a) a river, b) a lake, c) an estuary. Explain *in words and/or equations* how the *heat* equation is obtained. Why is it called the heat equation since we are solving a case of mass transfer?

10. Which is the order of the error of any centered approximation, and what is the order for backward and forward differences? What is the meaning of those "orders"? (Explain this in geometrical terms and with Taylor series expansions.)

11. If you have a method which is third-order accurate to solve an ODE, and you reduce the time step by half, how much the error is reduced? Please explain.

12. Mention four uses of water for which you can apply numerical techniques to solve specific problems. Please indicate the use, the problem, and how you will address it using the numerical techniques.

13. Draw a schematic of the Moody diagram and indicate clearly the zones. Indicate the parameters governing the behavior of each zone. What is the importance of the diagram?

14. What is the difference between an implicit and an explicit equation? Please give examples of each type of equation from the water resources field.

15. Please, take the equation:

$$\frac{d\,u(t)}{dt} + K\ u(t) = 0$$

In class we used a forward difference for the derivative to discretize it, and evaluated the second term explicitly (forward method). Now, please use a backward difference to discretize it and use the forward method. Call the time step as “h”. Is the solution unstable? Is the solution accurate? Under what circumstances?

16. Please, take the equation:

$$\frac{d\,u(t)}{dt} + K\ u(t) = 0$$

In class we used a forward difference for the derivative to discretize it, and evaluated the second term explicitly (forward method). Now, please use a Crank-Nicolson-type (centered) method. Call the time step as “h”. Is the solution unstable? Is the solution accurate? Under what circumstances?

17. Define consistency, stability and convergence for the solution of differential equations for a person that is non-technical. Give examples. Why do we need stability in addition to consistency in order to obtain convergence?

18. The Peclet number is: $U\ L/D$, where U is a characteristic velocity of the flow, L is a characteristic length of the flow, and D is the diffusion coefficient for a turbulent flow. Analyze the physical meaning of both cases in which the Peclet number is much larger than 1 and much lower than 1. Explain what the far and near fields are. Explain why the diffusion of any substance in a fluid is more important for a turbulent flow than for a laminar flow.

19. In which cases you can apply a compartment model to solve a problem? Explain what is the meaning and significance of a compartment model for a non-technical person. Which is the dimension of compartment models (0, 1, 2 or 3D)? Can you build 1D models from compartments?

20. Explain with graphs and words the meaning of 0D, 1D, 2D and 3D models. Please give examples of these models.

21. What steps do we take when we want to analyze a natural phenomenon? Think of analyzing the issue of flooding in the Yolo Bypass, starting from scratch.

22. Which type of equations are the following ones?

a) $Q^{j+1} = C_1\, I^{j+1} + C_1\, I^{j} + C_3\, Q^{j}$

b) $\frac{\partial h}{\partial t} = \frac{\partial}{\partial x}\left(D \frac{\partial h}{\partial x} \right) \quad ; h = h(x, t)$

c) $\frac{d h}{d t} = a\, h + b\, y \quad ; h = h(t);\ y = y(t)$

d) $x^4 + 23\, x^3 - 50\, x^2 - 4x + 6 = 0$

e) $h = \int_{t_0}^{t_1} D \frac{\partial h}{\partial x}\, dt \quad ; h = h(x, t)$

23. The equation of motion for a spherical particle moving horizontally through quiescent fluid of the same density can be approximated as:

$$\frac{d v_P}{d t} = -\frac{3}{4} C_D \frac{1}{D} v_p \left| v_p \right| \qquad (1)$$

where $\boldsymbol{v_p}$ is the particle velocity in the horizontal direction, $\boldsymbol{C_D}$ is the drag coefficient, t is time, and $\boldsymbol{D}$ is the particle diameter. For a certain particle size, considering that the fluid is water, the above equation can be rewritten as:

$$\frac{d v_P}{d t} = - K\, v_p \left| v_p \right| \qquad (2)$$

If the particle has an initial velocity, $v_p(0)$, write out the explicit (Euler), and an implicit formulation to solve numerically eq. (2). In the implicit case, consider the modulus of the velocity evaluated in the previous time step.

24. Explain the meaning of the Euler method to solve an ordinary differential equation. Which alternatives do you have to solve the same problem?

25. What do you understand by higher-order terms (H.O.Ts.)?

26. What is the meaning of "water resources"? Why water is so important? Describe 5 water-related problems that you know in United States.

27. Give an example of numerical instability. Why the problem of instability arises?

28. Develop a method of "unknown coefficients" or "undetermined coefficients" to determine the derivative of *second* order.

29. The equation of potential flow in two dimensions is given by (we did not see this in class, but you should know this from any basic Fluid Mechanics course):

$$\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} = 0$$

This equation is known as the Laplace equation, and ϕ indicates the flow potential. You are asked to discretize it, recalling that ϕ is the unknown function. The solution to this question involves 5 points in the computational molecule.

30. Describe a convenient compartment model for sediment transport in a river. What are the advantages of one such a model?

31. Develop a flow chart for the **Fixed-point** method for the solution of a general implicit equation.

32. Why do we say that the **Bisection** and **Regula-Falsi** methods are slow? How many iterations can be considered "slow"? How do these methods compare to the bracketing methods in terms of robustness?

33. Explain the physical meaning of the LAKE model. Which are the advantages and limitations of such a model? Would you apply the model to Clear Lake in California? Why or why not?

34. Describe the steps to follow in order to compute a backwater curve. Explain the procedure for a case in which you have to compute the water elevations and velocities in a river reach upstream of a dam.

35. What is the problem with the equation for backwater curves when you discretize it with a centered scheme? Why do we have two methods to compute the curves? Which is the advantage of one method over the other?

36. Define uniform flow and gradually varied flow. What is the difference between the equations for uniform flow and gradually varied flow, regarding type of equations and difficulties in solving them? Give an example of the computation of a backwater curve.

37. Explain the physical meaning of the equations in the model for BOD and dissolved oxygen. Why we need two coupled equations?

38. Explain the **Newton-Raphson** method applied to the solution of Backwater curves.

39. We saw in class that the equation for mass conservation in a river is given by:

$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = q_{lateral} \qquad (1)$$

which can be reworked as (we saw this in class, too):

$$\frac{\partial Q}{\partial x} + \alpha\ \gamma\ Q^{\gamma-1} \frac{\partial Q}{\partial t} = q_{lateral} \qquad (2)$$

We also saw that:

$$c_k = \frac{1}{\alpha\ \gamma\ Q^{\gamma-1}} \qquad (3)$$

Please, indicate first what is the meaning of $\boldsymbol{c_k}$. What does it represent?

40. a) Explain the basic concepts behind the Muskingum method, recalling that the Muskingum method is applied via the equation:

$$Q^{j+1} = C_1\ I^{j+1} + C_1\ I^{j} + C_3\ Q^{j}$$

b) When can you use the Muskingum method to route a flood? Which type of routing is it?

41. Explain in words how you route a flood in a river using the kinematic-wave approximation (write the equations and explain the meaning). Indicate how the final shape of the hydrograph relates to the incoming one.

42. When you solve numerically a “plug” flow in a channel for an initial condition of null concentrations, and a boundary condition of concentration equal to 1, what are the analytical and the numerical solutions? Which two “problems” you find in the numerical solution? Which types of discretization schemes are the best to solve those problems?

43. Solving numerically the diffusive part of the advection-diffusion equation, do you obtain a “propagation” of something? What does that propagation mean?

44. How does the concentration with both advection and diffusion look like, if the initial condition (distribution of concentration) is a square?

45. Define boundary and initial conditions. Which types of boundary conditions do you know?

46. Why the “space” coordinate does not appear in the Muskingum method equation?

47. You as an Engineer working for a private company, are kindly requested to compute the thickness of the flow (depth) of raining water moving on the pavement in a highway. You are given the intensity of the rain (in mm/hour). Please explain in words and graphs how you solve the problem, which theory you apply, which are the equations, the boundary conditions, and the expected results, if the slope of the highway is small. How do you solve the problem numerically?

48. Explain in words, graphs and equations how you would build a model for analyzing the concentration of dissolved oxygen and organic matter in a water body. Why these two variables go together?