

ECI 146- WATER RESOURCES SIMULATION

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PART I

Mass Balance of Nitrogen and Phosphorus

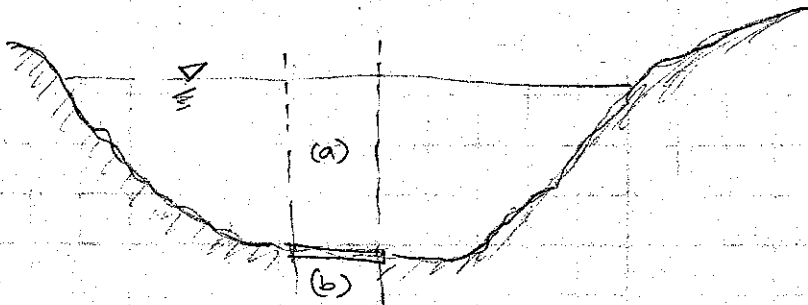
To examine the water quality of a lake, we used the LAKE model to perform a mass balance of phosphorus and nitrogen in a lake. The model is made with the assumption that the lake can be schematized as group of compartments. Two main compartments are established to represent the water column and the bed sediment.

The water compartment has both nitrogen and phosphorus's inputs (loadings) and output (outwashes); while the bed sediment compartment has demobilization. There are also other processes occurring in between the compartments; these include sedimentations and sediment release. In the special case of nitrogen, there is an addition chemical process of denitrification.

This section will illustrate and compute the concentration of nitrogen and phosphorus in each compartment with a time step of 0.02 years for year 5, 10, and 20. It also shows the differential equations involved in the Lake model calculation.

Lake Model

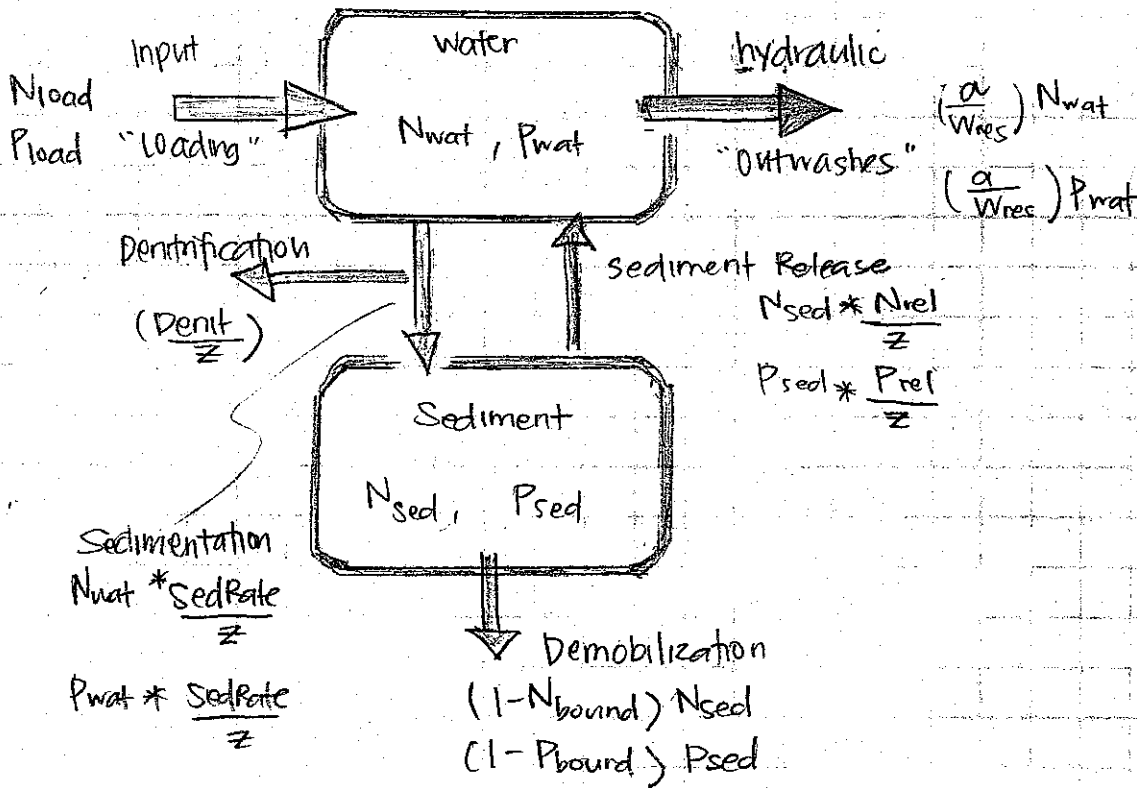
2 compartments: (a) the water column and (b) the bed sediment.



(a) $[N_{wat}, P_{wat}] = \text{mg/L}$

(b) $[N_{sed}, P_{sed}] = \text{g/m}^2$

Compartmentalizing



Nitrogen Submodel

$$1) \frac{dN_{wat}}{dt} = \frac{[(N_{load} - Denit) + N_{rel}N_{sed}]}{Z} - \frac{1}{W_{res}} N_{wat} * a - \frac{1}{Z} Sedrate * N_{wat}$$

$$2) \frac{dN_{sed}}{dt} = Sedrate * N_{wat} (1 - N_{bound}) - N_{rel} * N_{sed}$$

$$\text{Where: } Denit = N_{load} - 0.34W_{res}^{-0.16}Z^{0.17}$$

- Rearrange the Equations

$$1) \frac{dN_{wat}}{dt} = \frac{(N_{load} - Denit)}{Z} + \frac{N_{rel}}{Z} * N_{sed} - \left(\frac{a}{W_{res}} + \frac{Sedrate}{Z} \right) N_{wat}$$

$$2) \frac{dN_{sed}}{dt} = Sedrate(1 - N_{bound})N_{wat} - N_{rel}N_{sed}$$

- Substitute

$$\begin{array}{l} x = N_{wat} \\ y = N_{sed} \end{array} \quad \text{And} \quad \begin{array}{l} A = -\left(\frac{a}{W_{res}} + \frac{Sedrate}{Z} \right) \\ B = \frac{N_{rel}}{Z} \\ C = \left(\frac{N_{load} - Denit}{Z} \right) \end{array} \quad \begin{array}{l} D = Sedrate(1 - N_{bound}) \\ E = -N_{rel} \end{array}$$

- Implementing Euler Method: ODE (Ordinary Differential Equation)

1)

$$\frac{dX}{dt} = AX + BY + C$$

$$\frac{X_{j+1} - X_j}{\Delta t} = AX_j + BY_j + C$$

$$X_{j+1} = (AX_j + BY_j + C)\Delta t + X_j$$

2)

$$\frac{dY}{dt} = DX + EY$$

$$\frac{Y_{j+1} - Y_j}{\Delta t} = DX_j + EY_j$$

$$Y_{j+1} = (DX_j + EY_j)\Delta t + Y_j$$

Phosphorous Submodel

$$3) \frac{dP_{wat}}{dt} = \frac{[P_{load} + P_{rel}P_{sed}]}{Z} - \frac{1}{w_{res}}P_{wat}a - \frac{1}{Z}Sedrate * P_{wat}$$

$$4) \frac{dp_{sed}}{dt} = Sedrate * P_{wat} (1 - P_{bound}) - P_{rel}P_{sed}$$

- Rearrange the Equations

$$3) \frac{dP_{wat}}{dt} = \frac{P_{load}}{Z} + \left(\frac{P_{rel}}{Z}\right)P_{sed} - \left(\frac{a}{w_{res}} + \frac{Sedrate}{Z}\right)P_{wat}$$

$$4) \frac{dP_{sed}}{dt} = -P_{rel} * P_{sed} + Sedrate(1 - P_{bound})P_{wat}$$

- Substituting New Terms

$$U = P_{wat}$$

$$V = P_{sed}$$

$$F = -\left(\frac{a}{w_{res}} + \frac{Sedrate}{Z}\right)$$

$$G = \frac{P_{rel}}{Z}$$

$$H = \frac{P_{load}}{Z}$$

$$I = Sedrate(1 - P_{bound})$$

$$J = -P_{rel}$$

- Implementing Euler Method (ODE)

3)

$$\frac{dU}{dt} = FU + GV + H$$

$$\frac{U_{j+1} - U_j}{\Delta t} = FU_j + GV_j + H$$

$$U_{j+1} = (FU_j + GV_j + H)\Delta t + U_j$$

4)

$$\frac{dV}{dt} = IV + JV$$

$$\frac{V_{j+1} - V_j}{\Delta t} = IV_j + JV_j$$

$$V_{j+1} = (IV_j + JV_j)\Delta t + V_j$$

Example Calculations

$$X_{j+1} = (AX_j + BY_j + C)\Delta t + X_j$$

- Given Known Parameters

$$N_{load} = 25 \text{ g / m}^2 * \text{ yr}$$

$$N_{rel} = 0.9 / \text{ yr}$$

$$N_{bound} = 0.1$$

$$Sedrate = 30 \text{ m / yr}$$

$$w_{res} = 0.6 \text{ yr}$$

$$P_{load} = 1.6 \text{ g / m}^2 * \text{ yr}$$

$$P_{rel} = 0.8 / \text{ yr}$$

$$a = 1.0$$

Initial Boundary/Conditions

$$N_{wat} |_o = 4 \text{ mg / L} \Rightarrow X_j$$

$$N_{sed} |_o = 60 \text{ g / m}^2 \Rightarrow Y_j$$

$$P_{wat} |_o = 0.5 \text{ mg / L}$$

$$P_{sed} |_o = 15 \text{ g / m}^2$$

$$\Delta t = 0.02 \text{ yr}$$

- $$A = -\left(\frac{a}{w_{res}} + \frac{Sedrate}{Z}\right) = -\left(\frac{1.0}{0.6 \text{ yr}} + \frac{30 \text{ m / yr}}{1.8 \text{ m}}\right) = -18.33333 / \text{ yr}$$

- $$B = \frac{N_{rel}}{Z} = \frac{0.9 / \text{ yr}}{1.8 \text{ m}} = 0.5 / \text{ yr} * \text{ m}$$

- $$C = \left(\frac{N_{load} - Denit}{Z}\right) = \left(\frac{25 \text{ g / m}^2 * \text{ yr} - 24.59227 \text{ g / m}^2 * \text{ yr}}{1.8 \text{ m}}\right) = 0.226516 \text{ g / m} * \text{ yr}$$

Where:

$$Denit = N_{load} - 0.34w_{res}^{-0.16}Z^{0.17} = 25 \text{ g / m}^2 * \text{ yr} - 0.34(0.6 \text{ yr})^{-0.16}(1.8)^{0.17} = 24.59227 \text{ g / m}^2 * \text{ yr}$$

$$X_{j+1} = (-18.3333 / \text{ yr} * 4 \text{ mg / L} + 0.5 / \text{ yr} * \text{ m} * 60 \text{ g / m}^2 + 0.226516 \text{ g / m} * \text{ yr}) * 0.02 \text{ yr} + 4 \text{ mg / L}$$

$$X_{j+1} = 3.13786 \text{ mg / L}$$

- $$D = Sedrate(1 - N_{bound}) = 30 \text{ m / yr}(1 - 0.1) = 27 \text{ m / yr}$$

- $$E = -N_{rel} = -0.9 / \text{ yr}$$

$$Y_{j+1} = (DX_j + EY_j)\Delta t + Y_j \text{ [g / m}^2 \text{]}$$

$$Y_{j+1} = (27 \text{ m / yr} * 4 \text{ mg / L} - 0.9 / \text{ yr} * 60 \text{ g / m}^2)0.02 \text{ yr} + 60 \text{ g / m}^2 = 61.08 \frac{\text{g}}{\text{m}^2}$$

Known Parameters

		unit	
N_{load}	=	25	$g/m^2\text{-yr}$ Nitrogen input to the lake
N_{rel}	=	0.9	1/yr Sediment release rate of nitrogen
N_{bound}	=	0.1	Ratio of immobilized sedimentated nitrogen
z	=	1.8	m Mean depth of lake
SedRate	=	30	m/yr Mean sedimentation rate
W_{res}	=	0.6	yr Mean residence time of the water
P_{load}	=	1.6	$g/m^2\text{-yr}$ Phosphorus input to the lake
P_{rel}	=	0.8	1/yr Sediment release rate of phosphorus
P_{bound}	=	0.05	Ratio of immobilized sedimentated phosphorus
a	=	1	Correction factor of nutrient output due to thermocline formation

Unknown Variables

N_{wat}	=	?	mg/l Total nitrogen in water column
N_{sed}	=	?	g/m^2 Nitrogen in sediment
P_{wat}	=	?	mg/l Total phosphorus in water column
P_{sed}	=	?	g/m^2 Phosphorus in sediment

Initial Conditions

N_{wat_0}	=	4	mg/l
N_{sed_0}	=	60	g/m^2
P_{wat_0}	=	0.5	mg/l
P_{sed_0}	=	15	g/m^2

PART 1: EULER METHOD

$Denit = 24.592272$ (Denitrification) where

Nitrogen Submodel

(1) $dX/dt = AX + BY + C$
 (2) $dY/dt = DX + EY$

phosphorus Submodel

(3) $dU/dt = FU + GV + H$
 (4) $dV/dt = IU + JV$

X = N_{wat}
 Y = N_{sed}
 U = P_{wat}
 V = P_{sed}
 A = -18.3333333
 B = 0.5
 C = 0.22651582
 D = 27
 E = -0.9
 F = -18.3333333
 G = 0.44444444
 H = 0.88888889
 I = 28.5
 J = -0.8

Part 1 Results

Table 1

Δt (yrs)	Year	N_{wat}	N_{sed}	P_{wat}	P_{sed}
0.02	0	4	60	0.5	15
0.02	5	0.83509998	29.9260106	0.39107346	14.1236804
0.02	10	0.41696851	14.7260384	0.37654351	13.5274566
0.02	20	0.14019387	4.66468896	0.36288407	12.9672346

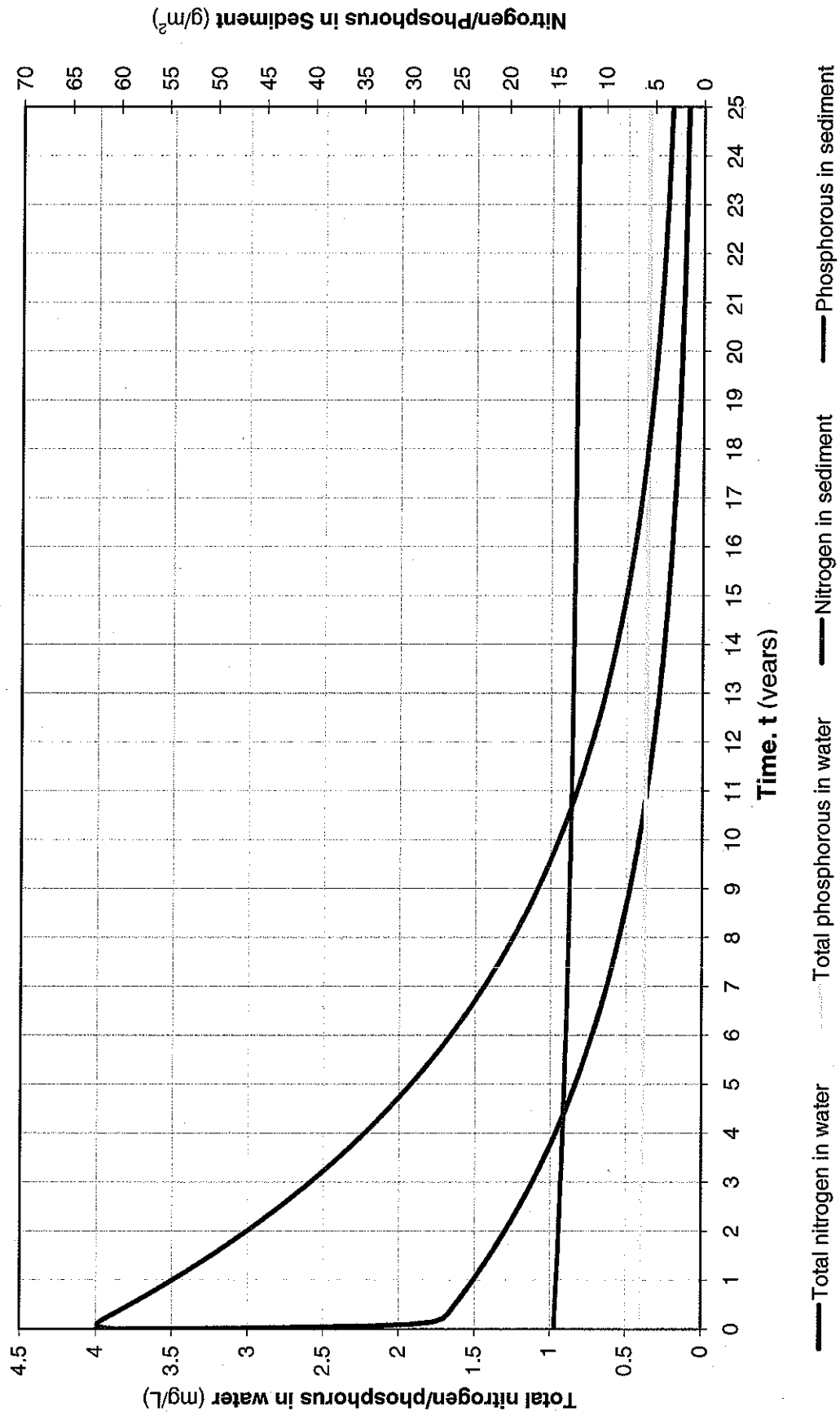
time step $\Delta t = 0.02$ yrs

Step by step calculations of the Lake Model using Euler Method.

Table 2

Time (yrs)	j	Nitrogen Submodel				Phosphorus Submodel			
		Nwat		Nsed		Pwat		Psed	
		X_j mg/L	X_{j+1} mg/L	Y_j g/m ²	Y_{j+1} g/m ²	U_j mg/L	U_{j+1} mg/L	V_j g/m ²	V_{j+1} g/m ²
0	0	4	3.13786365	60	61.08	0.5	0.46777778	15	15.045
0.02	1	3.13786365	2.60264396	61.08	61.6750064	0.46777778	0.44777037	15.045	15.0709133
0.04	2	2.60264396	2.26962156	61.6750064	61.970284	0.44777037	0.43532935	15.0709133	15.0850078
0.06	3	2.26962156	2.06166014	61.970284	62.0804145	0.43532935	0.42757533	15.0850078	15.0917854
0.08	4	2.06166014	1.93105255	62.0804145	62.0762635	0.42757533	0.42272469	15.0917854	15.0940348
0.1	5	1.93105255	1.8482929	62.0762635	62.0016592	0.42272469	0.41967261	15.0940348	15.0934833
0.12	6	1.8482929	1.79513241	62.0016592	61.8837075	0.41967261	0.41773473	15.0934833	15.091201
0.14	7	1.79513241	1.76028459	61.8837075	61.7391722	0.41773473	0.41648711	15.091201	15.0878506
0.16	8	1.76028459	1.73676894	61.7391722	61.5784208	0.41648711	0.41566718	15.0878506	15.0838426
0.18	9	1.73676894	1.72026819	61.5784208	61.4078645	0.41566718	0.41511226	15.0838426	15.0794314
0.2	10	1.72026819	1.70811215	61.4078645	61.2314677	0.41511226	0.4147216	15.0794314	15.0747745
0.22	11	1.70811215	1.69864935	61.2314677	61.0516819	0.4147216	0.41443279	15.0747745	15.0699694
0.24	12	1.69864935	1.69085839	61.0516819	60.8700223	0.41443279	0.41420716	15.0699694	15.0650766
0.26	13	1.69085839	1.68410752	60.8700223	60.6874254	0.41420716	0.41402077	15.0650766	15.0601334
0.28	14	1.68410752	1.678006	60.6874254	60.5044698	0.41402077	0.41385879	15.0601334	15.0551632
0.3	15	1.678006	1.67231215	60.5044698	60.3215126	0.41385879	0.41371201	15.0551632	15.05018
0.32	16	1.67231215	1.66687647	60.3215126	60.1387739	0.41371201	0.41357476	15.05018	15.045193
0.34	17	1.66687647	1.66160649	60.1387739	59.9563893	0.41357476	0.41344351	15.045193	15.0402075
0.36	18	1.66160649	1.65644498	59.9563893	59.7744418	0.41344351	0.41331607	15.0402075	15.035227
0.38	19	1.65644498	1.65135656	59.7744418	59.5929821	0.41331607	0.41319108	15.035227	15.0302536
0.4	20	1.65135656	1.64631929	59.5929821	59.412041	0.41319108	0.41306772	15.0302536	15.0252884
0.42	21	1.64631929	1.64131961	59.412041	59.2316366	0.41306772	0.41294545	15.0252884	15.0203324
0.44	22	1.64131961	1.6363491	59.2316366	59.0517798	0.41294545	0.41282396	15.0203324	15.015386
0.46	23	1.6363491	1.63140255	59.0517798	58.8724763	0.41282396	0.41270305	15.015386	15.0104495
0.48	24	1.63140255	1.62647669	58.8724763	58.6937291	0.41270305	0.41258259	15.0104495	15.005523
0.5	25	1.62647669	1.62156951	58.6937291	58.5155393	0.41258259	0.41246251	15.005523	15.0006067
0.52	26	1.62156951	1.61667973	58.5155393	58.3379072	0.41246251	0.41234276	15.0006067	14.9957007
0.54	27	1.61667973	1.61180655	58.3379072	58.1608319	0.41234276	0.41222331	14.9957007	14.9908048
0.56	28	1.61180655	1.60694945	58.1608319	57.9843125	0.41222331	0.41210414	14.9908048	14.9859192
0.58	29	1.60694945	1.60210809	57.9843125	57.8083475	0.41210414	0.41198524	14.9859192	14.9810439
0.6	30	1.60210809	1.59728225	57.8083475	57.6329357	0.41198524	0.4118666	14.9810439	14.9761788
0.62	31	1.59728225	1.59247177	57.6329357	57.4580752	0.4118666	0.41174821	14.9761788	14.9713239
0.64	32	1.59247177	1.58767652	57.4580752	57.2837646	0.41174821	0.41163008	14.9713239	14.9664792
0.66	33	1.58767652	1.58289643	57.2837646	57.1100022	0.41163008	0.4115122	14.9664792	14.9616446
0.68	34	1.58289643	1.57813141	57.1100022	56.9367862	0.4115122	0.41139457	14.9616446	14.9568203
0.7	35	1.57813141	1.5733814	56.9367862	56.764115	0.41139457	0.41127718	14.9568203	14.9520061
0.72	36	1.5733814	1.56864636	56.764115	56.5919869	0.41127718	0.41116005	14.9520061	14.947202
0.74	37	1.56864636	1.56392621	56.5919869	56.4204002	0.41116005	0.41104316	14.947202	14.942408
0.76	38	1.56392621	1.55922092	56.4204002	56.2493531	0.41104316	0.41092652	14.942408	14.937624
0.78	39	1.55922092	1.55453043	56.2493531	56.0788441	0.41092652	0.41081012	14.937624	14.9328502
0.8	40	1.55453043	1.5498547	56.0788441	55.9088713	0.41081012	0.41069396	14.9328502	14.9280863
0.82	41	1.5498547	1.54519367	55.9088713	55.7394332	0.41069396	0.41057806	14.9280863	14.9233325
0.84	42	1.54519367	1.54054731	55.7394332	55.5705279	0.41057806	0.41046239	14.9233325	14.9185887
0.86	43	1.54054731	1.53591556	55.5705279	55.402154	0.41046239	0.41034697	14.9185887	14.9138548
0.88	44	1.53591556	1.53129838	55.402154	55.2343096	0.41034697	0.41023179	14.9138548	14.9091309
0.9	45	1.53129838	1.52669572	55.2343096	55.0669932	0.41023179	0.41011685	14.9091309	14.9044169
0.92	46	1.52669572	1.52210754	55.0669932	54.900203	0.41011685	0.41000216	14.9044169	14.8997129
0.94	47	1.52210754	1.51753379	54.900203	54.7339374	0.41000216	0.4098877	14.8997129	14.8950187
0.96	48	1.51753379	1.51297442	54.7339374	54.5681948	0.4098877	0.40977349	14.8950187	14.8903344
0.98	49	1.51297442	1.5084294	54.5681948	54.4029734	0.40977349	0.40965952	14.8903344	14.8856599
1	50	1.5084294	1.50389867	54.4029734	54.2382718	0.40965952	0.40954578	14.8856599	14.8809953

Figure 1: Lake Model of Nitrogen and Phosphorus



PART II

Time Step Instability

To solve the Ordinary Differential Equations (ODE) of the Lake model, the Euler Method was used. The Euler approach provides good approximation of nitrogen and phosphorus concentrations if the chosen time step is small. Consequently, if the time step is too large it will make the solution become unstable. This section will demonstrate the value at which the Euler Method becomes ineffective in providing good approximations; this corresponds to the maximum time step value before the algorithm is unstable.

PART 2: VARYING THE TIME STEP

step $\Delta t = 0.04$ yrs

Time (yrs)	j	Nitrogen Submodel				Phosphorus Submodel			
		Nwat		Nsed		Pwat		Psed	
		X_j mg/L	X_{j+1} mg/L	Y_j g/m ²	Y_{j+1} g/m ²	U_j mg/L	U_{j+1} mg/L	V_j g/m ²	V_{j+1} g/m ²
0	0	4	2.2757273	60	62.16	0.5	0.43555556	15	15.09
0.04	1	2.2757273	1.85912125	62.16	62.3800255	0.43555556	0.41997037	15.09	15.1036533
0.08	2	1.85912125	1.75242681	62.3800255	62.1421955	0.41997037	0.41605705	15.1036533	15.0991026
0.12	3	1.75242681	1.71921836	62.1421955	61.7976974	0.41605705	0.41493259	15.0991026	15.0902364
0.16	4	1.71921836	1.70347281	61.7976974	61.4297361	0.41493259	0.41447512	15.0902364	15.080372
0.2	5	1.70347281	1.69191477	61.4297361	61.0580163	0.41447512	0.41417776	15.080372	15.0703017

step $\Delta t = 0.08$ yrs

Time (yrs)	j	Nitrogen Submodel				Phosphorus Submodel			
		Nwat		Nsed		Pwat		Psed	
		X_j mg/L	X_{j+1} mg/L	Y_j g/m ²	Y_{j+1} g/m ²	U_j mg/L	U_{j+1} mg/L	V_j g/m ²	V_{j+1} g/m ²
0	0	4	0.5514546	60	64.32	0.5	0.37111111	15	15.18
0.08	1	0.5514546	2.33357579	64.32	60.8801019	0.37111111	0.43765926	15.18	15.0546133
0.16	2	2.33357579	1.36432331	60.8801019	61.5372583	0.43765926	0.40214526	15.0546133	15.0889812
0.24	3	1.36432331	1.84292739	61.5372583	60.053514	0.40214526	0.41994043	15.0889812	15.0401776
0.32	4	1.84292739	1.56022905	60.053514	59.7103842	0.41994043	0.40990078	15.0401776	15.0350704
0.4	5	1.56022905	1.67842974	59.7103842	58.7813313	0.40990078	0.41440436	15.0350704	15.0073997

step $\Delta t = 0.16$ yrs

Time (yrs)	j	Nitrogen Submodel				Phosphorus Submodel			
		Nwat		Nsed		Pwat		Psed	
		X_j mg/L	X_{j+1} mg/L	Y_j g/m ²	Y_{j+1} g/m ²	U_j mg/L	U_{j+1} mg/L	V_j g/m ²	V_{j+1} g/m ²
0	0	4	-2.8970908	60	68.64	0.5	0.24222222	15	15.36
0.16	1	-2.8970908	11.1284847	68.64	46.2404077	0.24222222	0.76619259	15.36	14.4984533
0.32	2	11.1284847	-17.7795954	46.2404077	87.6568431	0.76619259	-0.30808233	14.4984533	16.1364895
0.48	3	-17.7795954	41.4226744	87.6568431	-1.77359425	-0.30808233	1.88533176	16.1364895	12.6661634
0.64	4	41.4226744	-80.1894821	-1.77359425	177.427757	1.88533176	-2.60204756	12.6661634	19.6420074
0.8	5	-80.1894821	169.263462	177.427757	-194.540403	-2.60204756	6.56961247	19.6420074	5.26249353

Time (yrs)	j	Nitrogen Submodel				Phosphorus Submodel			
		Nwat		Nsed		Pwat		Psed	
		X_j mg/L	X_{j+1} mg/L	Y_j g/m ²	Y_{j+1} g/m ²	U_j mg/L	U_{j+1} mg/L	V_j g/m ²	V_{j+1} g/m ²
0	0	4	-1.1728181	60	66.48	0.5	0.30666667	15	15.27
0.12	1	-1.1728181	5.42336362	66.48	55.5002294	0.30666667	0.55306667	15.27	14.85288
0.24	2	5.42336362	-3.15084069	55.5002294	67.0779027	0.55306667	0.23514027	14.85288	15.3184915
0.36	3	-3.15084069	7.83286488	67.0779027	49.6247654	0.23514027	0.64148456	15.3184915	14.652096
0.48	4	7.83286488	-6.39477004	49.6247654	69.643773	0.64148456	0.11833032	14.652096	15.439372
0.6	5	-6.39477004	11.8795323	69.643773	41.4031906	0.11833032	0.78810346	15.439372	14.361882

step $\Delta t = 0.10$ yrs

Time (yrs)	j	Nitrogen Submodel				Phosphorus Submodel			
		Nwat		Nsed		Pwat		Psed	
		X_j mg/L	X_{j+1} mg/L	Y_j g/m ²	Y_{j+1} g/m ²	U_j mg/L	U_{j+1} mg/L	V_j g/m ²	V_{j+1} g/m ²
0	0	4	-0.31068175	60	65.4	0.5	0.33888889	15	15.225
0.1	1	-0.31068175	3.55155304	65.4	58.6751593	0.33888889	0.48314815	15.225	14.9728333
0.2	2	3.55155304	-0.00321799	58.6751593	62.9835881	0.48314815	0.35172469	14.9728333	15.1519789
0.3	3	-0.00321799	3.17451265	62.9835881	57.3063766	0.35172469	0.46920626	15.1519789	14.9422359
0.4	4	3.17451265	0.24254321	57.3063766	60.7199869	0.46920626	0.36198304	14.9422359	15.0840949
0.5	5	0.24254321	2.85653159	60.7199869	55.9100547	0.36198304	0.45764057	15.0840949	14.909019

Time (yrs)	j	Nitrogen Submodel				Phosphorus Submodel			
		Nwat		Nsed		Pwat		Psed	
		X _j mg/L	X _{j,j} mg/L	Y _j g/m ³	Y _{j,j} g/m ³	U _j mg/L	U _{j,j} mg/L	V _j g/m ³	V _{j,j} g/m ³
0	0	4	0.12038642	60	64.86	0.5	0.355	15	15.2025
0.09	1	0.12038642	2.86083525	64.86	59.898879	0.355	0.45735	15.2025	15.018495
0.18	2	2.86083525	0.85629307	59.898879	61.9988995	0.45735	0.3834623	15.018495	15.1102661
0.27	3	0.85629307	2.25374641	61.9988995	59.0577808	0.3834623	0.43516015	15.1102661	15.0059077
0.36	4	2.25374641	1.21305139	59.0577808	59.7507043	0.43516015	0.39738221	15.0059077	15.0416682
0.45	5	1.21305139	1.92068471	59.7507043	57.8586121	0.39738221	0.42336829	15.0416682	14.9779534

Time (yrs)	j	Nitrogen Submodel				Phosphorus Submodel			
		Nwat		Nsed		Pwat		Psed	
		X _j mg/L	X _{j,j} mg/L	Y _j g/m ³	Y _{j,j} g/m ³	U _j mg/L	U _{j,j} mg/L	V _j g/m ³	V _{j,j} g/m ³
0	0	4	-0.09514766	60	65.13	0.5	0.34694444	15	15.21375
0.095	1	-0.09514766	3.18576185	65.13	59.3173312	0.34694444	0.46948565	15.21375	14.9968571
0.19	2	3.18576185	0.47631886	59.3173312	62.4171786	0.46948565	0.36944322	14.9968571	15.1282283
0.285	3	0.47631886	2.63306516	62.4171786	58.3022677	0.36944322	0.44918814	15.1282283	14.9787505
0.38	4	2.63306516	0.83802006	58.3022677	60.0712359	0.44918814	0.38373271	14.9787505	15.0565424
0.475	5	0.83802006	2.25337117	60.0712359	57.0846667	0.38373271	0.43556336	15.0565424	14.9512014

Table 3

Δt (yrs)	j	Actual	Year	Nwat	Nsed	Pwat	Psed
0.02	1	0.02	0.02	0.13786365	61.08	0.4677008	15.045
0.02	250	5	5	0.83509998	29.9260106	0.39107346	14.1225726
0.02	500	10	10	0.41696851	14.7260384	0.37652024	13.5245881
0.02	1000	20	20	0.13983518	4.66468896	0.36286782	12.9652349
0.04	1	5	0.04	2.2757273	62.16	0.43555556	15.09
0.04	125	5	5	0.83414591	29.8913281	0.39105376	14.1225726
0.04	250	10	10	0.41610094	14.6945003	0.37652024	13.5285018
0.04	500	20	20	0.13983518	4.65164963	0.36286782	12.9665684
0.08	1	0.08	0.08	0.5514546	64.32	0.37111111	15.18
0.08	63	5.04	5	0.82740627	29.6463278	0.39086484	14.1148243
0.08	125	10	10	0.4143613	14.6312607	0.37647358	13.5245881
0.08	250	20	20	0.13911862	4.6256012	0.36283531	12.9652349
0.16	1	0.16	0.16	-2.8970908	68.64	0.24222222	15.36
0.16	31	4.96	5	-1.0715E+10	1.5917E+10	-3.58367114	5.60310304
0.16	63	10.08	10	-1.0491E+20	1.5585E+20	-3.1133E+18	4.8677E+18
0.16	125	20	20	-2.3882E+39	3.5476E+39	-5.6214E+37	8.789E+37
0.10	1	0.1	0.10	0.31068175	65.4	0.33888889	15.225
0.10	50	5	5	0.84889711	29.7604696	0.39150273	14.1193434