UNIVERSITY OF CALIFORNIA, DAVIS
DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING

COURSE: WATER RESOURCES SIMULATION (ECI 146)

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COMPUTER PROBLEM 1: Solution of the Colebrook-White equation via three different methods.

Assigned on: Sunday, January 12, 2020
Due on: Monday, January 27, 2020

Problem 1

Please answer the following questions:

1) Please define explicit and implicit algebraic equations. What type of equations do you know? Please give examples of each type.
2) What is the physical meaning of *smooth* and *fully-rough* regimes in pipes? In which area of the Moody chart is the Colebrook-White equation valid?
3) Please discuss the difference between the “true” and “approximate” errors in the bisection method. Why do we use the latter if these two errors are different? In other words, are we safe or not by using the approximate error?
4) Which method is more robust? Which method is more accurate? What do we mean by robust? What do we mean by accurate?

Problem 2

Please develop a flow chart for the bisection method used to calculate the friction factor for a pipe in any regime.

Problem 3. Introduction

The Moody chart is the most reliable source of information for the computation of flow resistance (i.e., head losses) in pipes and, to some extent, in open channels. Based on extensive sets of measurements and regressions, it gives the Darcy friction factor, f, as a function of the Reynolds number, and the ratio between the equivalent roughness and the pipe diameter. The diagram presents several regions for *turbulent* flow, as follows:

a) a zone of "fully-rough" flow towards the right side of the diagram, where the *relative roughness* determines the characteristics of the flow, and in which the Reynolds number does not play any role in determining the flow behavior;
b) a zone of smooth behavior, in which the roughness value is immaterial in defining the flow characteristics, and
c) an intermediate zone, usually called "transition zone," where both variables are important in characterizing the flow.

On the left, the *laminar* flow region appears, which is a function of the Reynolds number.

The use of the diagram is straightforward: we must follow the curve pertaining to the value of the "relative roughness" (from the right of the diagram) until we find the vertical corresponding to the Reynolds number (computed as the product of the mean-flow velocity and the pipe diameter, divided by the kinematic viscosity of water). Notice that the Reynolds number is given in logarithmic scale. Then, we exit the diagram horizontally, starting from the point defined above, towards the left of the diagram.

The Colebrook-White formula was proposed in the first half of last century (1939), and allows for the *computation of* f. It is expressed as follows:

$$\frac{1}{\sqrt{f}} = -2 \, log_{10} \left(\frac{\varepsilon}{3.7\, D} + \frac{2.51}{\Re \sqrt{f}} \right) \tag{1}$$

where ε or k indicates the pipe roughness, D is the pipe diameter, and $\Re$ is the Reynolds number. The main characteristic of Equation (1) is that it is *implicit*. It can be observed that the Darcy friction factor (White, 2011; page 356) appears at both sides in (1), and there is no way of obtaining it solely as a function of known variables. Therefore, a numerical method needs to be applied to compute that coefficient. Equation (1) can be expressed as:

$$\frac{1}{\sqrt{f}} + 0.869 \, ln\left(\frac{\varepsilon}{3.7\, D} + \frac{2.51}{\Re \sqrt{f}} \right) = 0 \tag{2}$$

where the logarithm on base 10 has been converted to the "natural" logarithm. Multiplying by $\sqrt{f}$, Equation (2) yields:

$$F = 1 + \sqrt{f} \; 0.869 \, ln\left(\frac{\varepsilon}{3.7\, D} + \frac{2.51}{\Re \sqrt{f}} \right) = 0 \tag{3}$$

which is still an implicit equation.

Tasks

You are asked to:

1. Please, develop a code for the **bisection** method to obtain the value of f in the following **11** points, well distributed on the Moody diagram: $\Re$=3 x 10^6- ε/D =0.0008; $\Re$=3 x 10^6- ε/D=0.00005; $\Re$=3 x 10^7- ε/D=0.00001; $\Re$=3 x 10^7- ε/D =0.002; $\Re$=3 x 10^7- ε/D=0.015; $\Re$=3 x 10^5- ε/D=0.002; $\Re$=3 x 10^5- ε/D=0.03; $\Re$=3 x 10^4- ε/D=0.002; $\Re$=3 x 10^4- ε/D=0.01; $\Re$=3 x 10^5- ε/D=10^{-10}; $\Re$=3 x 10^2- ε/D=10^{-10}. Start the iterations with an interval [0, 0.1], and determine the number of iterations needed to achieve a tolerance of 10^{-2}. To stop the computations, please compare two successive values of f, as follows:

$$\frac{|f_k - f_{k-1}|}{f_k} 100 < tolerance \qquad (4)$$

2. Indicate the regimes to which the above points pertain (fully rough, smooth or transition), by determining where the points fall on the Moody chart.
3. Please, redo part 1. using the **fixed-point**, or iteration-of-a-point method (also called Picard method), for the first five points in 1. Determine the number of iterations needed to achieve a convergence of 10^{-2}, defined in the same way as in point 1. Start with a guessed value of f =0.05.
4. Compare your results in 1. and 3. with the following *explicit* formula proposed by Swammee and Jain (1976):

$$f = \frac{0.25}{\left[-\log_{10}\left(\frac{\varepsilon}{3.7\,D} + \frac{5.76}{(\Re)^{0.9}}\right)\right]^2} \qquad (5)$$

   and with the following *explicit* formula by Haaland:

$$f^{-1/2} = -1.8\log_{10}\left(\left(\frac{\varepsilon}{3.7\,D}\right)^{1.1} + \frac{6.9}{(\Re)}\right) \qquad (6)$$

5. Please, redo part 1. using the **Newton-Raphson** method. For this part, it is better to solve for $X = 1/\sqrt{f}$ (this will be your new variable; start from Eq. (2), not with Eq. (1), and re-arrange it in terms of X), and then to back-calculate the resistance coefficient f for each iteration; this will ease the analytical determination of the derivative. As an initial guess, please start with f =0.05. Please, determine the number of iterations needed to achieve a tolerance of 10^{-2}, defined in the same way as in point 1., **using** f **, not** X .
6. Please re-do computations of part 1., using tolerances of 10^{-1}, 10^{-3}, and 10^{-4} with the bisection method.

7. Please, compare the results obtained with all the methods and discuss similarities and differences (what you expect to obtain and what you finally obtain). You can present results with a table, for example.

The more you discuss, the more points you obtain in your grade for this computer problem. You have to present in the report **the code you developed** in the language you selected, the comparisons and results and, very importantly, **the discussion of results**. Please be neat and clear in your presentation.

Note: Keep in mind that D is the *internal* diameter (ID) of the pipe. Very often, the *nominal* diameter does not agree with the ID.

Problem 4

This problem is associated with the flow in a pipe connecting two reservoirs. The fluid moving in the pipe has a kinematic viscosity of 2 $\times 10^{-5}$ m^2/s, the pipe diameter is 0.3 m, and the pipe length is 100 m. The head loss, in turn, is 8 m, and the relative roughness height is 0.0002. In ECI 141, we saw that this problem could be solved with the method proposed by Rouse in closed form, or with an iterative method using the Moody chart. The aim of this problem is to compute the velocity of the flow, the Darcy friction factor, f, and the Reynolds number by using the iteration-of-a-point method (fixed-point method) **for two equations**.

To that end:

a) Please write down the Bernoulli equation for the flow in the pipe, taking one point at the free surface of each reservoir.
b) Please write down the Colebrook-White equation. You will have two implicit equations in v and f.
c) Arrange the two equations in such a way that you can obtain one variable from each equation.
d) Apply the fixed-point method to the system of two equations with two unknowns. You can start the iterations with a value of velocity equal to 5 m/s and f_0=0.002. Stop the iterations when you reach a tolerance of 10^{-4}, as defined in Problem 1.
e) Please show the equations, the code you developed, the iterations, the values of the velocity of the flow and f, the Reynolds number, and your final result (values and number of iterations).
f) Please assess the response of the method to three different initial guesses of your choice. Please discuss your results.