situation. The following is a list of the major factors that should be considered in this selection process:

(1) Backwater effects. Backwater effects can be produced by tidal fluctuations, significant tributary inflows, dams, bridges, culverts, and channel constrictions. A floodwave that is subjected to the influences of backwater will be attenuated and delayed in time. Of the hydrologic methods discussed previously, only the modified puls method is capable of incorporating the effects of backwater into the solution. This is accomplished by calculating a storage-discharge relationship that has the effects of backwater included in the relationship. Storage-discharge relationships can be determined from steady flow-water surface profile calculations, observed water surface profiles, normal depth calculations, and observed inflow and outflow hydrographs. All of these techniques, except the normal depth calculations, are capable of including the effects of backwater into the storage-discharge relationship. Of the hydraulic methods discussed in this chapter, only the kinematic wave technique is not capable of accounting for the influences of backwater on the floodwave. This is due to the fact that the kinematic wave equations are based on uniform flow assumptions and a normal depth downstream boundary condition.

(2) Floodplains. When the flood hydrograph reaches a magnitude that is greater than the channels carrying capacity, water flows out into the overbank areas. Depending on the characteristics of the overbanks, the flow can be slowed greatly, and often ponding of water can occur. The effects of the floodplains on the floodwave can be very significant. The factors that are important in evaluating to what extent the floodplain will impact the hydrograph are the width of the floodplain, the slope of the floodplain in the lateral direction, and the resistance to flow due to vegetation in the floodplain. To analyze the transition from main channel to overbank flows, the modeling technique must account for varying conveyance between the main channel and the overbank areas. For 1-D flow models, this is normally accomplished by calculating the hydraulic properties of the main channel and the overbank areas separately, then combining them to formulate a composite set of hydraulic relationships. This can be accomplished in all of the routing methods discussed previously except for the Muskingum method. The Muskingum method is a linear routing technique that uses coefficients to account for hydrograph timing and diffusion. These coefficients are usually held constant during the routing of a given floodwave. While these coefficients can be calibrated to match the peak flow and timing of a specific flood magnitude, they can not be used to model a range of floods that may remain in bank or go out of bank. When modeling floods through extremely flat and wide floodplains, the assumption of 1-D flow in itself may be inadequate. For this flow condition, velocities in the lateral direction (across the floodplain) may be just as predominant as those in the longitudinal direction (down the channel). When this occurs, a two-dimensional (2-D) flow model would give a more accurate representation of the physical processes. This subject is beyond the scope of this chapter. For more information on this topic, the reader is referred to EM 1110-2-1416.

(3) Channel slope and hydrograph characteristics. The slope of the channel will not only affect the velocity of the floodwave, but it can also affect the amount of attenuation that will occur during the routing process. Steep channel slopes accelerate the floodwave, while mild channel slopes are prone to slower velocities and greater amounts of hydrograph attenuation. Of all the routing methods presented in this chapter, only the complete unsteady flow equations are capable of routing floodwaves through channels that range from steep to extremely flat slopes. As the channel slopes become flatter, many of the methods begin to break down. For the simplified hydraulic methods, the terms in the momentum equation that were excluded become more important in magnitude as the channel slope is decreased. Because of this, the range of applicable channel slopes decreases with the number of terms excluded from the momentum equation. As a rule of thumb, the kinematic wave equations should only be applied to relatively steep channels (10 ft/mile or greater). Since the diffusion wave approximation includes the pressure differential term in the momentum equation, it is applicable to a wider range of slopes than the kinematic wave equations. The diffusion wave technique can be used to route slow rising floodwaves through extremely flat slopes. However, rapidly rising floodwaves should be limited to mild to steep channel slopes (approximately 1 ft/mile or greater). This limitation is due to the fact that the acceleration terms in the momentum equation increase in magnitude as the time of rise of the inflowing hydrograph is decreased. Since the diffusion wave method does not include these acceleration terms, routing rapidly rising hydrographs through flat channel slopes can result in errors in the amount of diffusion that will occur. While "rules of thumb" for channel slopes can be established, it should be realized that it is the combination of channel slope and the time of rise of the inflow hydrograph together that will determine if a method is applicable or not.

(a) Ponce and Yevjevich (1978) established a numerical criteria for the applicability of hydraulic routing techniques. According to Ponce, the error due to the use of the kinematic wave model (error in hydrograph peak accumulated after an elapsed time equal to the hydrograph duration) is within 5 percent, provided the following inequality is satisfied:

$$\frac{TS_o u_o}{d_o} \geq 171 \tag{9-45}$$

where

T = hydrograph duration, in seconds

S_o = friction slope or bed slope

u_o = reference mean velocity

d_o = reference flow depth

When applying Equation 9-45 to check the validity of using the kinematic wave model, the reference values should correspond as closely as possible to the average flow conditions of the hydrograph to be routed.

(b) The error due to the use of the diffusion wave model is within 5 percent, provided the following inequality is satisfied:

$$TS_o\left(\frac{g}{d_o}\right)^{1/2} \geq 30 \tag{9-46}$$

where g = acceleration of gravity. For instance, assume S_o = 0.001, u_o = 3 ft/s, and d_o = 10 ft. The kinematic wave model will apply for hydrographs of duration larger than 6.59 days. Likewise, the diffusion wave model will apply for hydrographs of duration larger than 0.19 days.

(c) Of the hydrologic methods, the Muskingum-Cunge method is applicable to the widest range of channel slopes and inflowing hydrographs. This is due to the fact that the Muskingum-Cunge technique is an approximation of the diffusion wave equations, and therefore can be applied to channel slopes of a similar range in magnitude. The other hydrologic techniques use an approximate relationship in place of the momentum equation. Experience has shown that these techniques should not be applied to channels with slopes less than 2 ft/mi. However, if there is gauged data available, some of the parameters of the hydrologic methods can be calibrated to produce the desired attenuation effects that occur in very flat streams.

(4) Flow networks. In a dendritic stream system, if the tributary flows or the main channel flows do not cause significant backwater at the confluence of the two streams, any of the hydraulic or hydrologic routing methods can be applied. If significant backwater does occur at the confluence of two streams, then the hydraulic methods that can account for backwater (full unsteady flow and diffusion wave) should be applied. For full networks, where the flow divides and possibly changes direction during the event, only the full unsteady flow equations and the diffusion wave equations can be applied.

(5) Subcritical and supercritical flow. During a flood event, a stream may experience transitions between subcritical and supercritical flow regimes. If the supercritical flow reaches are long, or if it is important to calculate an accurate stage within the supercritical reach, the transitions between subcritical and supercritical flow should be treated as internal boundary conditions and the supercritical flow reach as a separate routing section. This is normally accomplished with hydraulic routing methods that have specific routines to handle supercritical flow. In general, none of the hydrologic methods have knowledge about the flow regime (supercritical or subcritical), since hydrologic methods are only concerned with flows and not stages. If the supercritical flow reaches are short, they will not have a noticeable impact on the discharge hydrograph. Therefore, when it is only important to calculate the discharge hydrograph, and not stages, hydrologic routing methods can be used for reaches with small sections of supercritical flow.

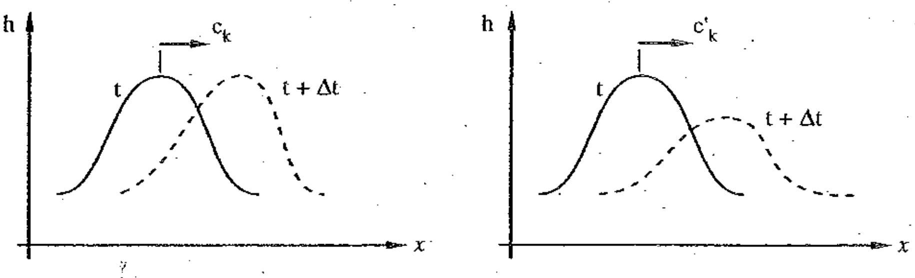
(6) Observed data. In general, if observed data are not available, the routing methods that are more physically based are preferred and will be easier to apply. When gauged data are available, all of the methods should be calibrated to match observed flows and/or stages as best as possible. The hydraulic methods, as well as the Muskingum-Cunge technique, are considered physically based in the sense that they only have one parameter (roughness coefficient) that must be estimated or calibrated. The other hydrologic methods may have more than one parameter to be estimated or calibrated. Many of these parameters, such as the Muskingum X and the number of subreaches (NSTPS), are not related directly to physical aspects of the channel and inflowing hydrograph. Because of this, these methods are generally not used in ungauged situations. The final choice of a routing model is also influenced by other factors, such as the required accuracy, the type and availability of data, the type of information desired (flow hydrographs, stages, velocities, etc.), and the familiarity and experience of the user with a given method. The modeler must take all of these factors

into consideration when selecting an appropriate routing technique for a specific problem. Table 9-3 contains a list of some of the factors discussed previously, along with some guidance as to which routing methods are appropriate and which are not. This table should be used as guidance in selecting an appropriate method for routing discharge hydrographs. By no means is this table all inclusive.

Table 9-3
Selecting the Appropriate Channel Routing Technique

Factors to consider in the selection of a routing technique.	Methods that are appropriate for this specific factor.	Methods that are not appropriate for this factor.
1. No observed hydrograph data available for calibration.	* Full Dynamic Wave * Diffusion Wave * Kinematic Wave * Muskingum-Cunge	* Modified Puls * Muskingum * Working R&D
2. Significant backwater that will influence discharge hydrograph.	* Full Dynamic Wave * Diffusion Wave * Modified Puls * Working R&D	* Kinematic Wave * Muskingum * Muskingum-Cunge
3. Flood wave will go out of bank into the flood plains.	* All hydraulic and hydrologic methods that calculate hydraulic properties of main channel separate from overbanks.	* Muskingum
4. Channel slope > 10 ft/mile and $\frac{TS_o u_o}{d_o} \geq 171$	* All methods presented	* None
5. Channel slopes from 10 to 2 ft/mile and $\frac{TS_o u_o}{d_o} < 171$	* Full Dynamic Wave * Diffusion Wave * Muskingum-Cunge * Modified Puls * Muskingum * Working R&D	* Kinematic Wave
6. Channel slope < 2 ft/mile and $TS_o\left(\frac{g}{d_o}\right)^{1/2} \geq 30$	* Full Dynamic Wave * Diffusion Wave * Muskingum-Cunge	* Kinematic Wave * Modified Puls * Muskingum * Working R&D
7. Channel slope < 2 ft/mile and $TS_o\left(\frac{g}{d_o}\right)^{1/2} < 30$	* Full Dynamic Wave	* All others

HAND OUT 21: Kinematic wave (Chapter 6 of our syllabus). Sources: Altinakar, M., and Graf, W. (1998). *"Fluvial Hydraulics."* John Wiley and Sons



a) *onde cinématique*   b) *onde diffusive*

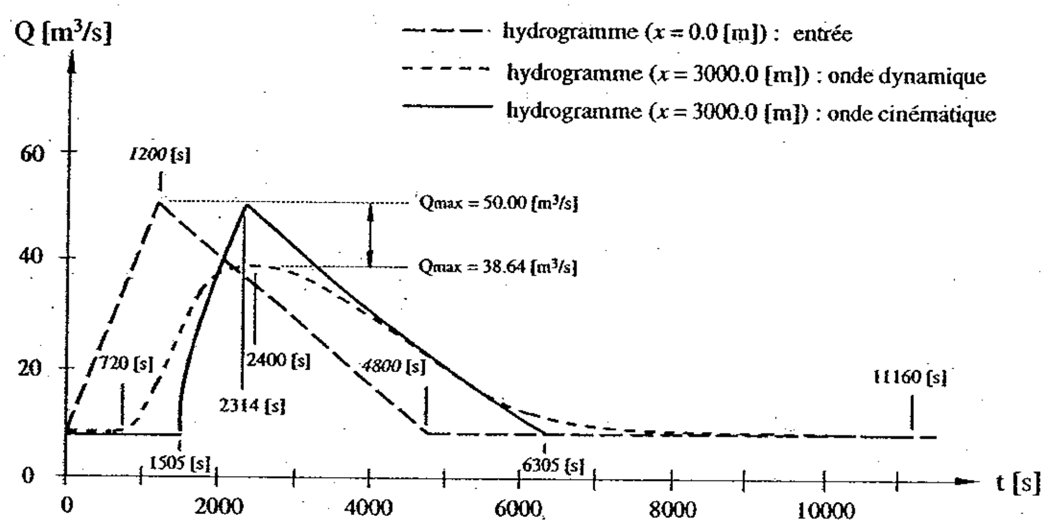
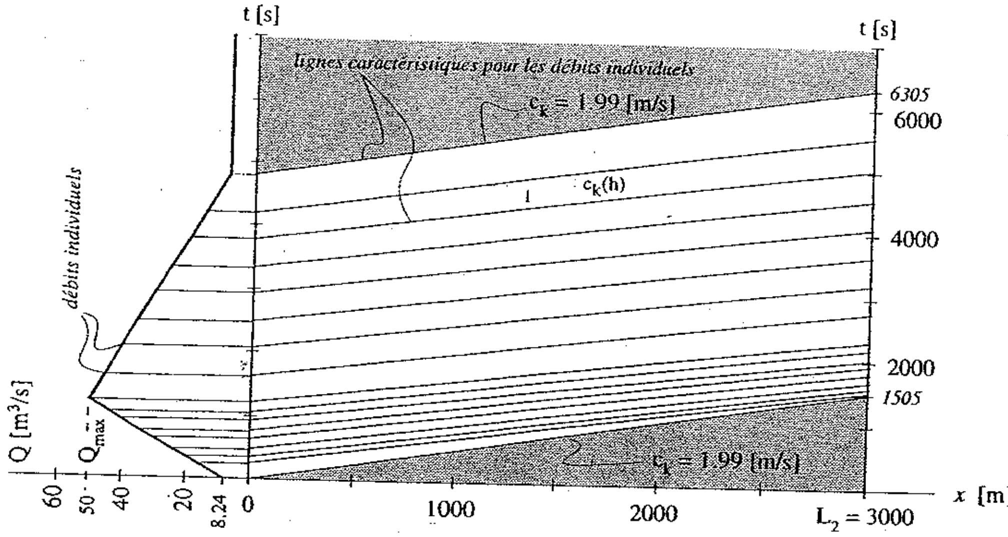
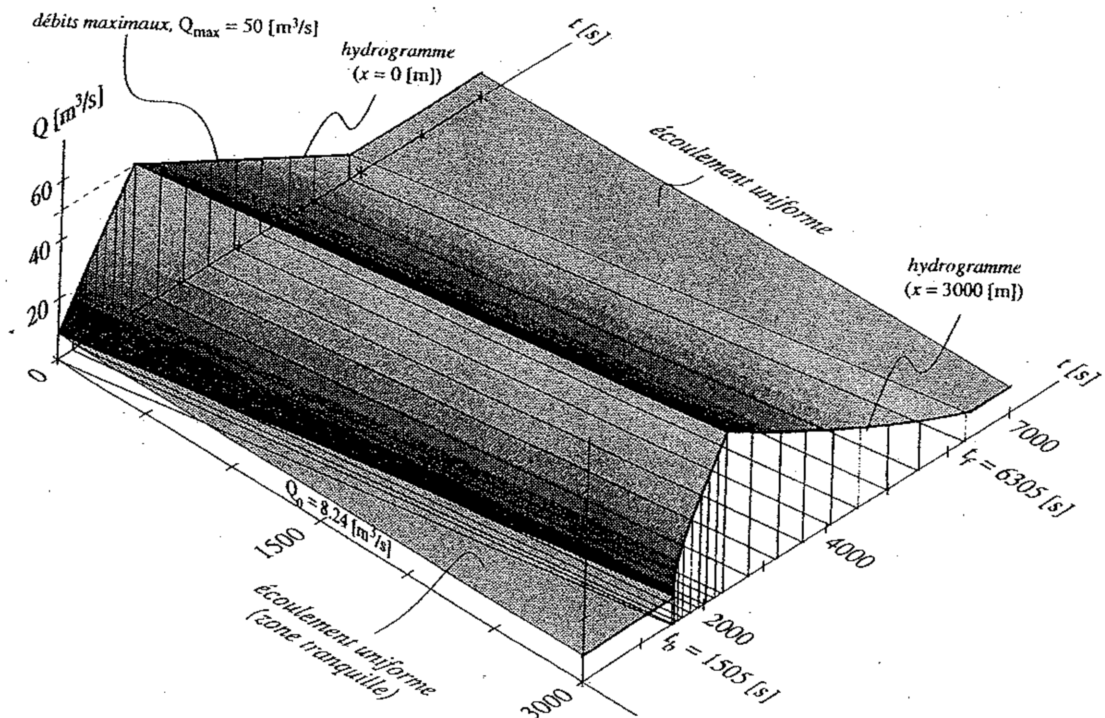


Fig. Ex.5.C.3
Comparaison des hydrogrammes : onde cinématique et onde dynamique.


t [s]
t [s]
lignes caractéristiques pour les débits individuels
c_k = 1.99 [m/s]
6305
6000
$c_k(h)$
4000
débits individuels
2000
1505
Q [m³/s]
Q_{max}
c_k = 1.99 [m/s]
x [m]
60
50
40
20
8.24
0
1000
2000
L_2 = 3000


débits maximaux, Q_{max} = 50 [m³/s]
hydrogramme
(x = 0 [m])
t [s]
Q [m³/s]
60
40
20
0
écoulement uniforme
hydrogramme
(x = 3000 [m])
t [s]
7000
t_f = 6305 [s]
4000
Q_0 = 8.24 [m³/s]
1500
2000
t_b = 1505 [s]
écoulement uniforme
(zone tranquille)
L_2 = 3000
x [m]

HAND OUT 22: Continuous source of a pollutant (Chapter 7 of our syllabus). Sources: Altinakar, M., and Graf, W. (1998). *"Fluvial Hydraulics."* John Wiley and Sons, and Koutitas, C. G. (1983). "Elements of computational hydraulics." Pentech Press, UK.

In the absence of velocity, $\vec{V} = 0$, eq. 8.35 becomes evidently eq. 8.32.

2° For one-dimensional convection-diffusion in the x-direction, in a channel with a weak velocity, $u \equiv U$, being uniformly distributed over the flow depth, the above equation, eq. 8.35, can be written as :

$$C(x,t) = \frac{\mathcal{M}_1}{\sqrt{4\pi\,\varepsilon_m t}} \exp\left[-\frac{(x - Ut)^2}{4\,\varepsilon_m t}\right] \tag{8.36}$$

where C is the average concentration in a section, S, of the channel.

The total mass of the substance, M_0, introduced instantaneously and uniformly over the section, S, defines :

$$\mathcal{M}_1 = \frac{M_0}{S} = \int_{-\infty}^{+\infty} C(x,t)\,dx = \int_{-\infty}^{+\infty} C(x,0)\,dx \tag{8.27a}$$

3° All remarks made for pure diffusion (see sect. 8.2.1) remain valid; the velocity of translation, u or U, is taken into account by coordinate transformation, $x' = x - ut$ or $x' = x - Ut$ (see Fig. 8.3 and Fig. 8.6).

The mass, M_0, displaces itself with the velocity of translation, U, and at the same time it spreads out according to the normal curve (see Fig. 8.6). The maximum concentration, C_{max}, is propagated with the velocity and it decreases with time.

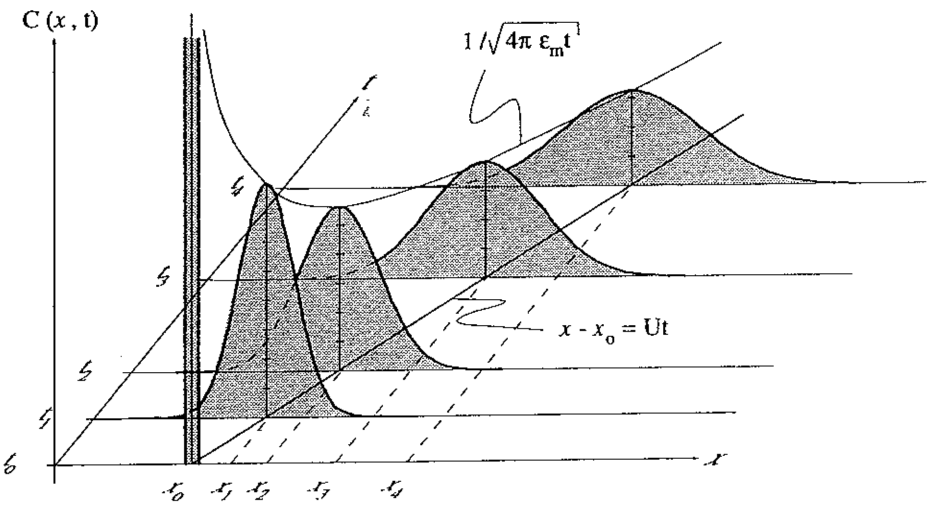


Fig. 8.6 Evolution of the concentration, $C(x,t)$, for a mass, M_0, injected instantaneously at $x = x_0$ into a medium in motion, U.

8.3.2 Continuous Source

1° Considered will be the one-dimensional convection-diffusion in a medium moving with a non-zero velocity, $\vec{V}(u,0,0) \neq 0$.

At a certain station, $x = 0$, an average concentration is introduced in a continuous and constant way, $C_o = Cte$.

The average velocity, U, being weak (without distribution over the flow depth) transports the average concentration, C, and diffusion takes place at the same time.

2° The solution to the one-dimensional convection-diffusion equation (see eq. 8.6b) is (see *Daily* et *Harleman*, 1966, p. 434) given by :

$$C(x,t) = \frac{C_o}{2}\left[\exp\left(\frac{Ux}{\varepsilon_m}\right) erfc\left(\frac{x+Ut}{\sqrt{4\,\varepsilon_m t}}\right) + erfc\left(\frac{x-Ut}{\sqrt{4\,\varepsilon_m t}}\right)\right] \tag{8.37}$$

In the absence of velocity, U = 0, eq. 8.37 becomes evidently eq. 8.33.

The evolution of the concentration, C(x,t), is shown at Fig. 8.7. Note that the concentration of the value $C_o/2$ displaces itself with the velocity of the flow, U.

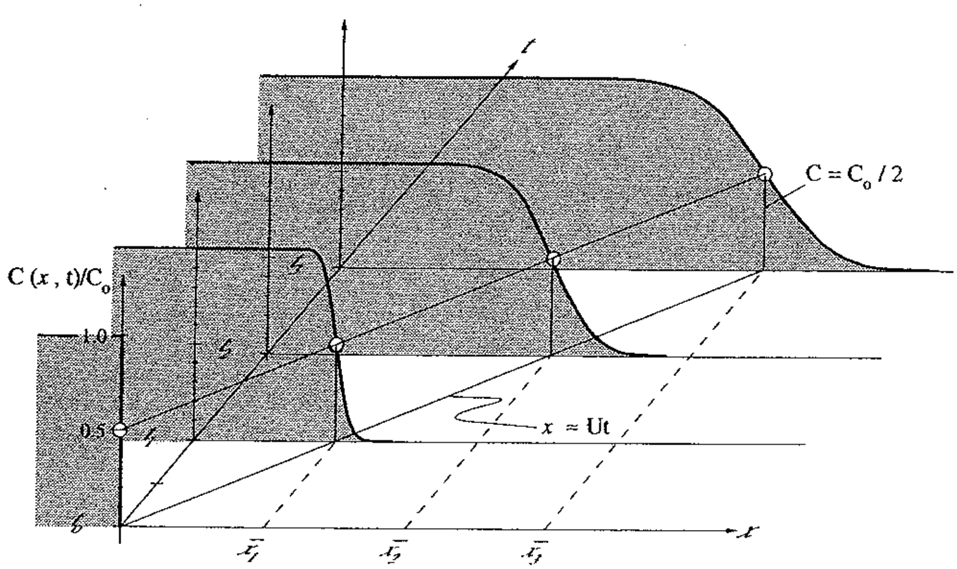


Fig. 8.7 Evolution of the concentration, C(x,t), for a concentration, C_o, introduced continuously into a flow with an average velocity, U.

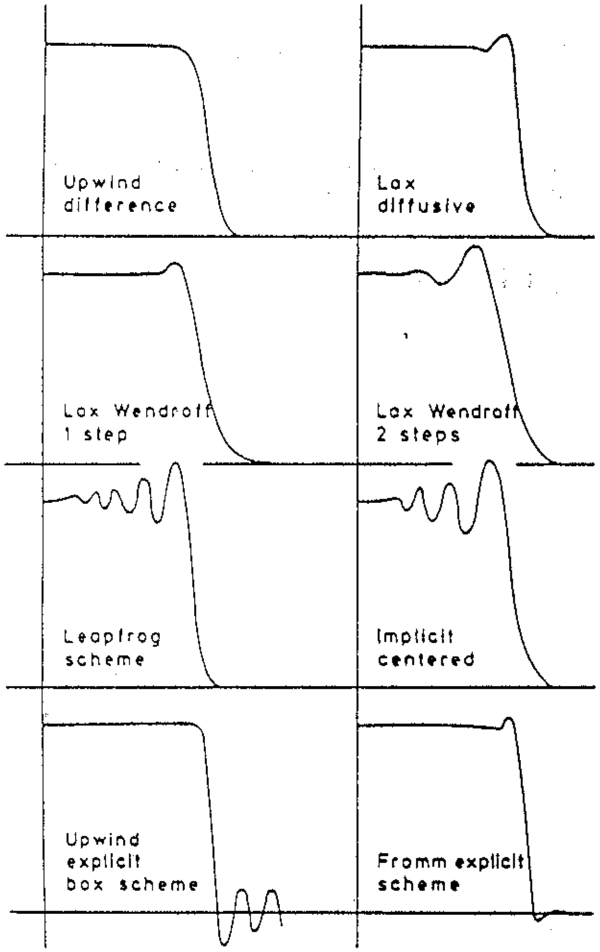


Fig. 3.1 1-D advection equation. Solutions by various finite difference schemes

+ 1, at a point i, j of the flow domain, has the form

$$c_{ij}^{n+1} = c_{ij}^{n} - \frac{(\Delta t)u}{2\Delta x}(c_{i+1j}^{n} - c_{i-1j}^{n}) - \frac{(\Delta t)v}{2\Delta y}(c_{ij+1}^{n} - c_{ij-1}^{n})$$
$$+ \frac{(\Delta t)D_x}{\Delta x^2}(c_{i+1j}^{n} - 2c_{ij}^{n} + c_{i-1j}^{n})$$
$$+ \frac{(\Delta t)D_y}{\Delta y^2}(c_{ij+1}^{n} - 2c_{ij}^{n} + c_{ij-1}^{n}) \qquad (3.22)$$

(It should be noted that centered differences for pure advection are unstable and the diffusion part of the operator is necessary to suppress instability.) The explicit scheme is very easily programmable but the time step Δt is limited by the known stability criteria. The use of implicit finite differences for both parts of the operators $u\,\partial c/\partial x$, $D\,\partial^2 c/\partial x^2$ lead to an algebraic system in IM × JM unknowns (where IM, JM are the maximum values of the indices I, J along x, y respectively). The fully implicit scheme leads to the inductive relation

$$\frac{c_{ij}^{n+1} - c_{ij}^{n}}{\Delta t} = -u\left(\frac{c_{i+1j}^{n+1} - c_{i-1j}^{n+1}}{2\Delta x}\right) - v\left(\frac{c_{ij+1}^{n+1} - c_{ij-1}^{n+1}}{2\Delta y}\right)$$
$$+ \frac{D_x(c_{i+1j}^{n+1} - 2c_{ij}^{n+1} + c_{i-1j}^{n+1})}{\Delta x^2}$$
$$+ \frac{D_y(c_{ij+1}^{n+1} - 2c_{ij}^{n+1} + c_{ij+1}^{n+1})}{\Delta y^2} \qquad (3.23)$$

In order to avoid the large computational volume required for the fully implicit scheme (retaining at the same time the qualifications of the implicitness) it is preferable to apply the ADI technique (Alternating Directions Implicit). According to this method the solution advances from time level n to $n+1$ using an implicit scheme along y and explicit along x, while alternately from $n+1$ to $n+2$, using an implicit scheme along x and explicit along y. The inductive formula takes the forms:

$$c_{ij}^{n+1} = c_{ij}^{n} - \frac{u\Delta t}{2\Delta x}(c_{i+1j}^{n} - c_{i-1j}^{n}) - \frac{v\Delta t}{2\Delta y}(c_{ij+1}^{n+1} - c_{ij-1}^{n+1})$$
$$+ \frac{(\Delta t)D_x}{\Delta x^2}(c_{i+1j}^{n} - 2c_{ij}^{n} + c_{i-1j}^{n})$$
$$+ \frac{(\Delta t)D_y}{\Delta y^2}(c_{ij+1}^{n+\frac{1}{2}} - 2c_{ij}^{n+\frac{1}{2}} + c_{ij-1}^{n+\frac{1}{2}}) \qquad (3.24)$$

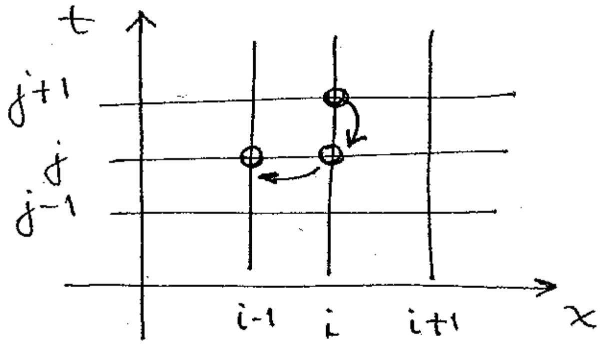
$$c_{ij}^{n+2} = c_{ij}^{n+1} - \frac{u\Delta t}{2\Delta x}(c_{i+1j}^{n+2} - c_{i-1j}^{n+2}) - \frac{v\Delta t}{2\Delta y}(c_{ij+1}^{n+1} - c_{ij-1}^{n+1})$$
$$+ \frac{(\Delta t)D_x}{\Delta x^2}(c_{i+1j}^{n+\frac{3}{2}} - 2c_{ij}^{n+\frac{3}{2}} + c_{i-1j}^{n+\frac{3}{2}})$$
$$+ \frac{(\Delta t)D_y}{\Delta y^2}(c_{ij+1}^{n+1} - 2c_{ij}^{n+1} + c_{ij-1}^{n+1}) \qquad (3.25)$$

This technique leads to the solution of IM algebraic systems in JM unknowns when advancing from n to $n+1$ and to the solution of JM

HAND OUT 23: Numerical schemes to solve the advection and diffusion equations (Chapter 7 of our syllabus).

SCHEMES

1) UPWIND:



$$\frac{C_i^{j+1} - C_i^j}{\Delta t} = -U\frac{(C_i^j - C_{i-1}^j)}{\Delta x}$$

$$\Rightarrow \boxed{C_i^{j+1} = C_i^j\left[1 - U\frac{\Delta t}{\Delta x}\right] + \frac{\Delta t}{\Delta x} U C_{i-1}^j}$$

2) LEAP FROG

$$\frac{C_i^{j+1} - C_i^{j-1}}{2\,\Delta t} = -U\frac{(C_{i+1}^j - C_{i-1}^j)}{2\,\Delta x}$$

$$\boxed{C_i^{j+1} = -\frac{\Delta t}{\Delta x} U (C_{i+1}^j - C_{i-1}^j) + C_i^{j-1}}$$

3) LAX

$$\frac{C_i^{j+1} - \dfrac{(C_{i+1}^j + C_{i-1}^j)}{2}}{\Delta t} = -\frac{U}{2\Delta x}(C_{i+1}^j - C_{i-1}^j)$$

4) DIFFUSION:

$$\frac{C_i^{j+1} - C_i^j}{\Delta t} = D\left[\frac{C_{i+1}^j - 2C_i^j + C_{i-1}^j}{\Delta x^2}\right]$$

$$C_i^{j+1} = C_i^j \left[1 - 2\mathcal{D}\right] + \mathcal{D}\left[C_{i+1}^j + C_{i-1}^j\right]$$

where $\mathcal{D} = D\dfrac{\Delta t}{\Delta x^2}$