Table 9-1
Storage Routing Calculation

(1)	(2)	(3)	(4)	(5)	(6)	(7)
Time (hr)	Inflow (cfs)	Average Inflow (cfs)	$\frac{S}{\Delta t} + \frac{O}{2}$ (cfs)	Outflow (cfs)	$\frac{S}{\Delta t}$ (cfs)	S (acre-ft)
0	3,000		8,600	3,000	7,100	1,760
		3,130				
3	3,260		8,730	3,150	7,155	1,774
		3,445				
6	3,630		9,025	3,400	7,325	1,816
		3,825				
9	4,020		9,450	3,850	7,525	1,866
		4,250				
12	4,480		9,850	4,300	7,700	1,909
.						
.						
etc.						

the corresponding discharge at the outlet. If channel or levee modifications will have an effect on the routing through the reach,modifications can be made to the cross sections, water surface profiles recalculated, and a revised storage-outflow relationship can be developed. The impacts of the channel or levee modification can be approximated by routing floods with both pre- and post-project storage-outflow relationships.

(c) Observed water surface profiles, obtained from high water marks, can be used to compute storage-outflow relationships. Sufficient stage data over a range of floods are required for this type of calculation; however, it is not likely that enough data would be available over the range of discharges needed to compute an adequate storage discharge relationship. If a few observed profiles are available, they can be used to calibrate a steady-flow water surface profile model for the channel reach of interest. Then the water surface profile model could be used to calculate the appropriate range of values to calculate the storage-outflow relationship.

(d) Normal depth associated with uniform flow does not exist in natural streams; however, the concept can be used to estimate water depth and storage in natural rivers if uniform flow conditions can reasonably be assumed. With a typical cross section, Manning's equation is solved for a range of discharges, given appropriate "n" values and an estimated slope of the energy grade line. Under the assumption of uniform flow conditions, the energy slope is considered equal to the average channel bed slope; therefore, this approach should not be applied in backwater areas.

(e) Observed inflow and outflow hydrographs can be used to compute channel storage by an inverse process of flood routing. When both inflow and outflow are known, the change in storage can be computed, and from that a storage versus outflow function can be developed. Tributary inflow, if any, must also be accounted for in this calculation. The total storage is computed from some base level storage at the beginning or end of the routing sequence.

(f) Inflow and outflow hydrographs can also be used to compute routing criteria through a process of iteration in which an initial set of routing criteria is assumed, the inflow hydrograph is routed, and the results are evaluated. The process is repeated as necessary until a suitable fit of the routed and observed hydrograph is obtained.

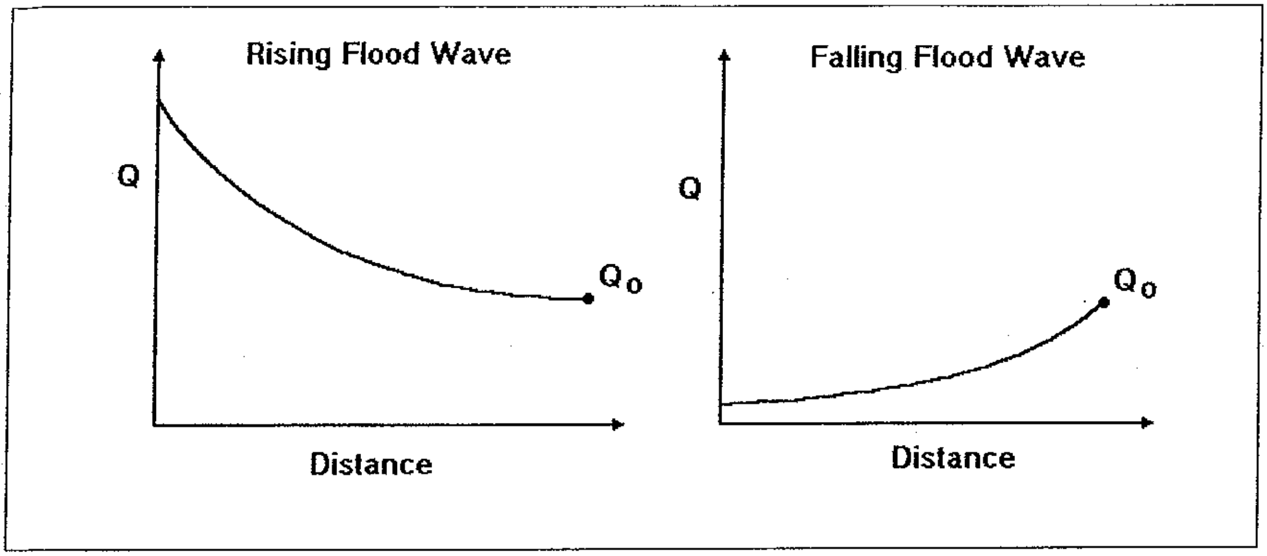

Figure 9-6. Rising and falling floodwave

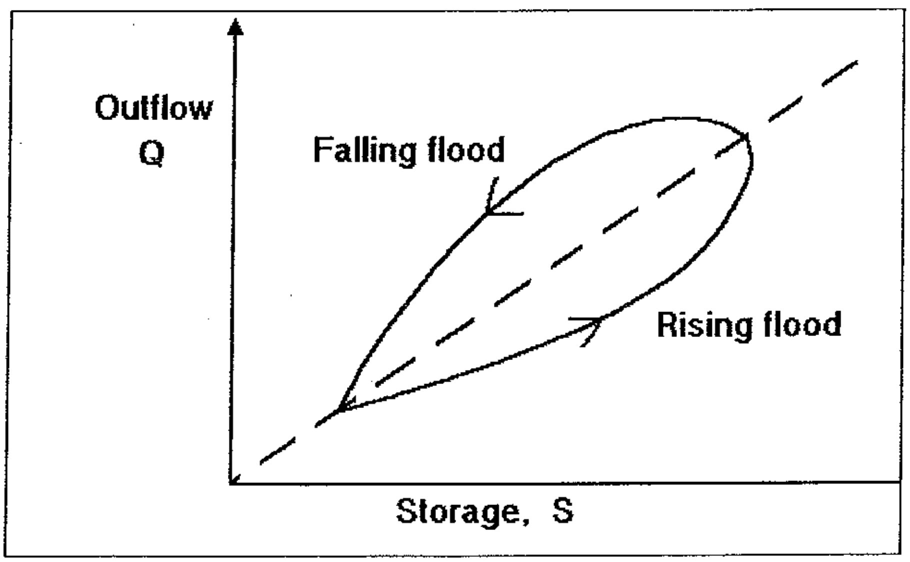

Figure 9-7. Looped storage-outflow relationship for a river reach

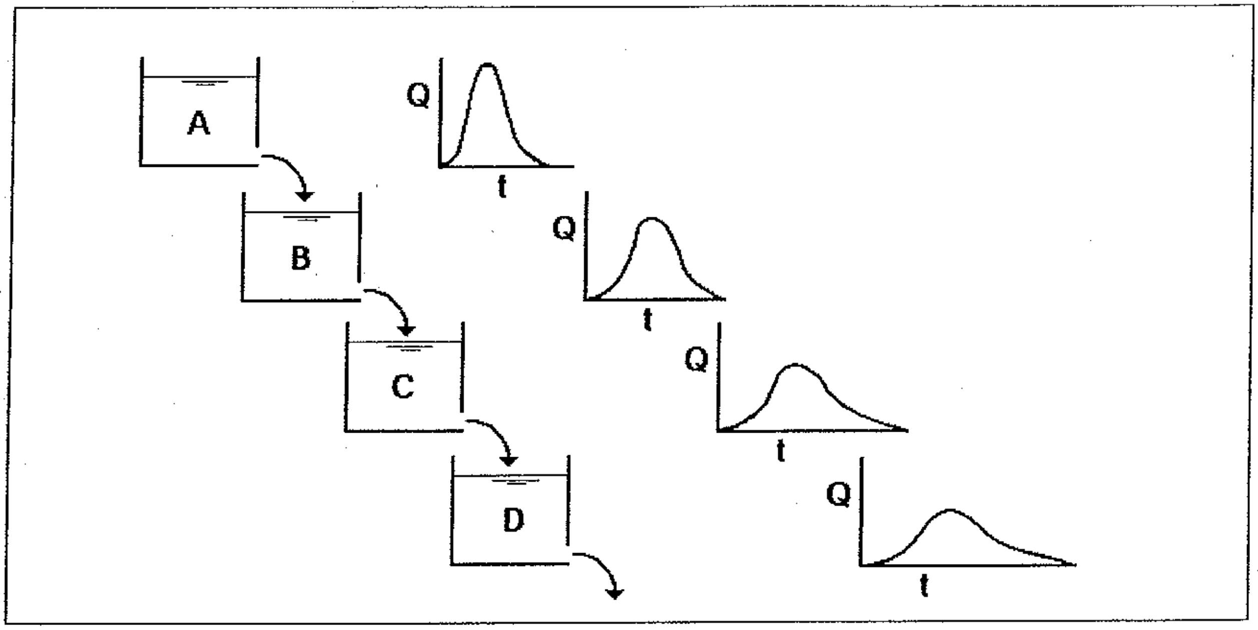

Figure 9-8. Cascade of reservoirs, depicting storage routing in a channel

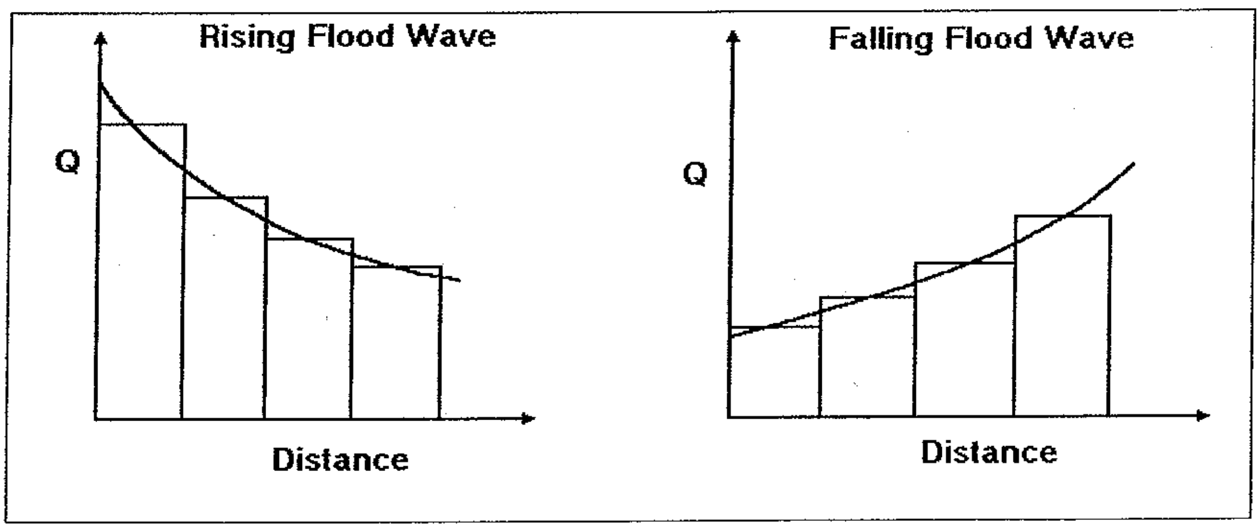

Figure 9-9. Modified puls approximation of the rising and falling floodwaves

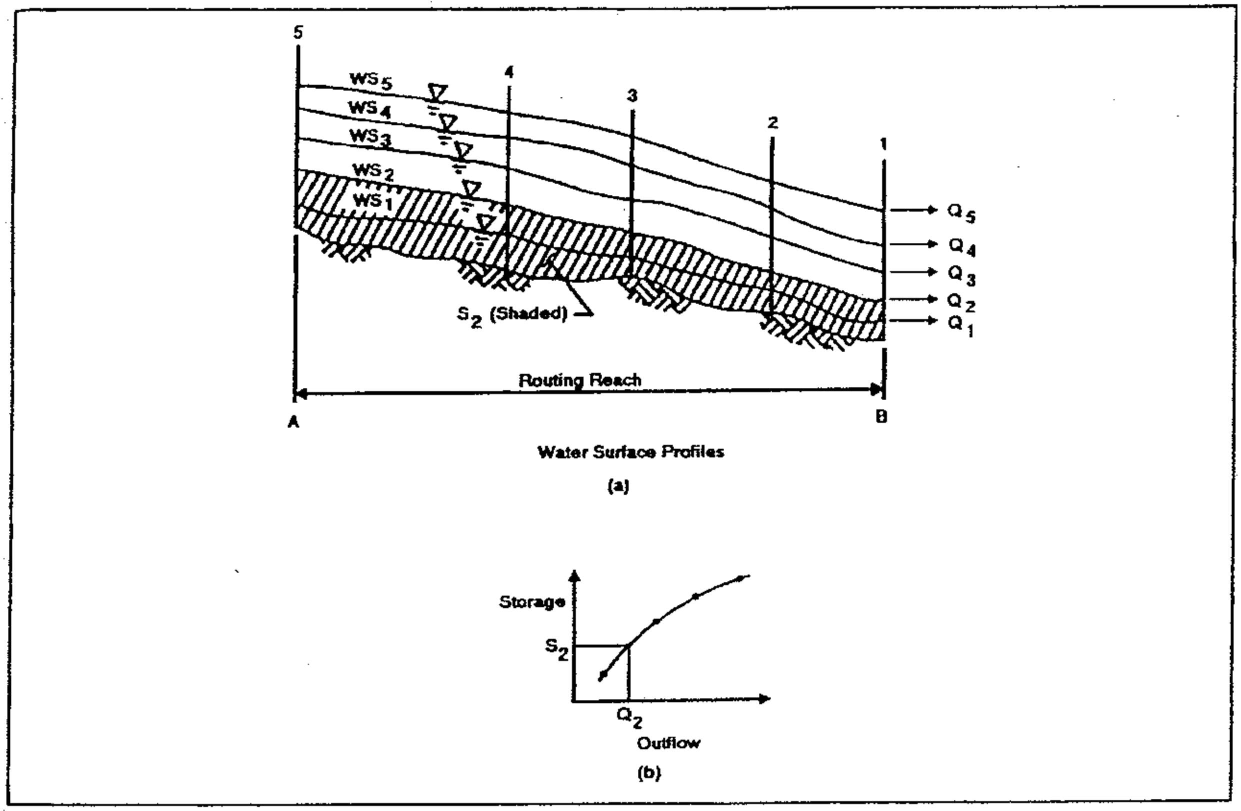

Figure 9-10. Storage-outflow relationships

(3) Determining the number of routing steps. In reservoir routing, the modified puls method is applied with one routing step. This is under the assumption that the travel time through the reservoir is smaller than the computation interval Δt. In channel routing, the travel time through the river reach is often greater than the computation interval. When this occurs, the channel must be broken down into smaller routing steps to simulate the floodwave movement and changes in hydrograph shape. The number of steps (or reach lengths) affects the attenuation of the hydrograph and should be obtained by calibration. The maximum amount of attenuation will occur when the channel routing computation is done in one step. As the number of routing steps increases, the amount of attenuation decreases. An initial estimate of the number of routing steps (NSTPS) can be obtained by dividing the total travel time (K) for the reach by the computation interval Δt.

$$K = \frac{L}{V_w}$$

$$NSTPS = \frac{K}{\Delta t} \tag{9-13}$$

where

K = floodwave travel time through the reach

L = channel reach length

V_w = velocity of the floodwave (not average velocity)

$NSTPS$ = number of routing steps

The time interval Δt is usually determined by ensuring that there is a sufficient number of points on the rising side of the inflow hydrograph. A general rule of thumb is that the computation interval should be less than 1/5 of the time of rise (t_r) of the inflow hydrograph.

$$\Delta t \leq \frac{t_r}{5} \qquad (9\text{-}14)$$

c. Muskingum method. The Muskingum method was developed to directly accommodate the looped relationship between storage and outflow that exists in rivers. With the Muskingum method, storage within a reach is visualized in two parts: prism storage and wedge storage. Prism storage is essentially the storage under the steady-flow water surface profile. Wedge storage is the additional storage under the actual water surface profile. As shown in Figure 9-11, during the rising stages of the floodwave the wedge storage is positive and added to the prism storage. During the falling stages of a floodwave, the wedge storage is negative and subtracted from the prism storage.

(1) Development of the Muskingum routing equation.

(a) Prism storage is computed as the outflow (O) times the travel time through the reach (K). Wedge storage is computed as the difference between inflow and outflow (I-O) times a weighting coefficient X and the travel time K. The coefficient K corresponds to the travel time of the floodwave through the reach. The parameter X is a dimensionless value expressing a weighting of the relative effects of inflow and outflow on the storage (S) within the reach. Thus, the Muskingum method defines the storage in the reach as a linear function of weighted inflow and outflow:

$$S = \text{prism storage} + \text{wedge storage}$$

$$S = KO + KX(I-O)$$

$$S = K\,[XI + (1-X)O] \qquad (9\text{-}15)$$

where

S = total storage in the routing reach

O = rate of outflow from the routing reach

I = rate of inflow to the routing reach

K = travel time of the floodwave through the reach

X = dimensionless weighting factor, ranging from 0.0 to 0.5

(b) The quantity in the brackets of Equation 9-15 is considered an expression of weighted discharge. When $X = 0.0$, the equation reduces to $S = KO$, indicating that storage is only a function of outflow, which is equivalent to level-pool reservoir routing with storage as a linear function of outflow. When $X = 0.5$, equal weight is given to inflow and outflow, and the condition is equivalent to a uniformly progressive wave that does not attenuate. Thus, "0.0" and "0.5" are limits on the value of X, and within this range the value of X determines the degree of attenuation of the floodwave as it passes through the routing

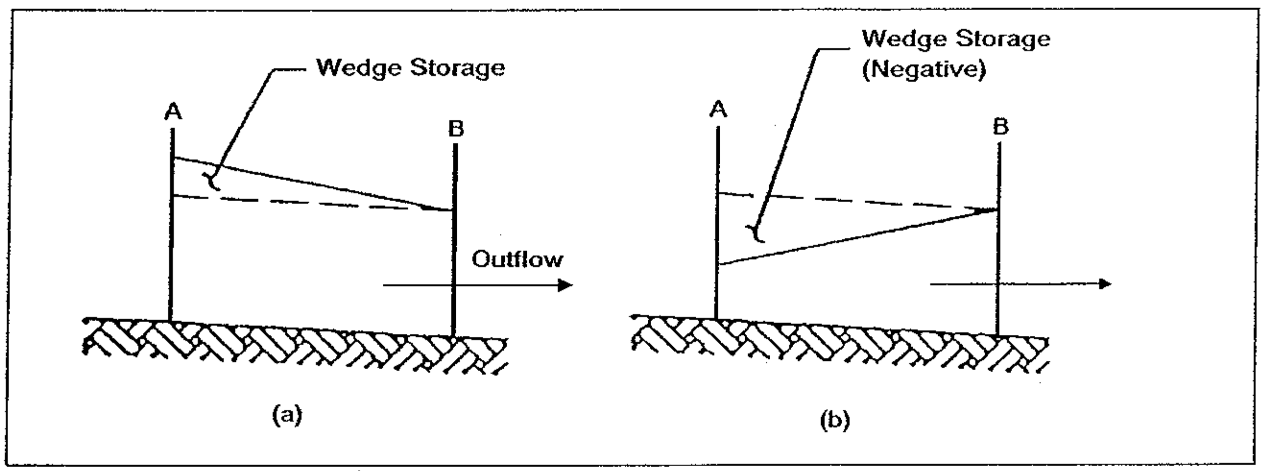

Figure 9-11. Muskingum prism and wedge storage concept

reach. A value of "0.0" produces maximum attenuation, and "0.5" produces pure translation with no attenuation.

(c) The Muskingum routing equation is obtained by combining Equation 9-15 with the continuity equation, Equation 9-11, and solving for O_2.

$$O_2 = C_1 I_2 + C_2 I_1 + C_3 O_1 \qquad (9\text{-}16)$$

The subscripts 1 and 2 in this equation indicate the beginning and end, respectively, of a time interval Δt. The routing coefficients C_1, C_2, and C_3 are defined in terms of Δt, K, and X.

$$C_1 = \frac{\Delta t - 2KX}{2K(1 - X) + \Delta t} \qquad (9\text{-}17)$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1 - X) + \Delta t} \qquad (9\text{-}18)$$

$$C_3 = \frac{2K(1 - X) - \Delta t}{2K(1 - X) + \Delta t} \qquad (9\text{-}19)$$

Given an inflow hydrograph, a selected computation interval Δt, and estimates for the parameters K and X, the outflow hydrograph can be calculated.

(2) Determination of Muskingum K and X. In a gauged situation, the Muskingum K and X parameters can be calculated from observed inflow and outflow hydrographs. The travel time, K, can be estimated as the interval between similar points on the inflow and outflow hydrographs. The travel time of the routing reach can be calculated as the elapsed time between centroid of areas of the two hydrographs, between the hydrograph peaks, or between midpoints of the rising limbs. After K has been estimated, a value for X can be obtained through trial and error. Assume a value for X, and then route the inflow hydrograph with these parameters. Compare the routed hydrograph with the observed outflow hydrograph. Make adjustments to X to obtain the desired fit. Adjustments to the original estimate of K may also be necessary to obtain the best overall fit between computed and observed hydrographs. In an ungauged situation, a value for K can be estimated as the travel time of the floodwave through the routing reach. The floodwave velocity (V_w) is greater than the average velocity at a given cross section for a given discharge. The floodwave velocity can be estimated by a number of different techniques:

(a) Using Seddon's law, a floodwave velocity can be approximated from the discharge rating curve at a station whose cross section is representative of the routing reach. The slope of the discharge rating curve is equal to dQ/dy. The floodwave velocity, and therefore the travel time K, can be estimated as follows:

$$V_w = \frac{1}{B}\frac{dQ}{dy} \qquad (9\text{-}20)$$

$$K = \frac{L}{V_w} \qquad (9\text{-}21)$$

where

V_w = floodwave velocity, in feet/second

B = top width of the water surface

L = length of the routing reach, in feet

(b) Another means of estimating floodwave velocity is to estimate the average velocity (V) and multiply it by a ratio. The average velocity can be calculated from Manning's equation with a representative discharge and cross section for the routing reach. For various channel shapes, the floodwave velocity has been found to be a direct ratio of the average velocity.

Channel shape	Ratio V_w/V
Wide rectangular	1.67
Wide parabolic	1.44
Triangular	1.33

For natural channels, an average ratio of 1.5 is suggested. Once the wave speed has been estimated, the travel time (K) can be calculated with Equation 9-21.

(c) Estimating the Muskingum X parameter in an ungauged situation can be very difficult. X varies between 0.0 and 0.5, with 0.0 providing the maximum amount of hydrograph attenuation and 0.5 no attenuation. Experience has shown that for channels with mild slopes and flows that go out of bank, X will be closer to 0.0. For steeper streams, with well defined channels that do not have flows going out of bank, X will be closer to 0.5. Most natural channels lie somewhere in between these two limits, leaving a lot of room for "engineering judgment." One equation that can be used to estimate the Muskingum X coefficient in ungauged areas has been developed by Cunge (1969). This equation is taken from the Muskingum-Cunge channel routing method, which is described in paragraph 9-3*e*. The equation is written as follows:

$$X = \frac{1}{2}\left(1 - \frac{Q_o}{BS_o c\Delta x}\right) \tag{9-22}$$

where

Q_o = reference flow from the inflow hydrograph

c = floodwave speed

S_o = friction slope or bed slope

B = top width of the flow area

Δx = length of the routing subreach

The choice of which flow rate to use in this equation is not completely clear. Experience has shown that a reference flow based on average values (midway between the base flow and the peak flow) is in general the most suitable choice. Reference flows based on peak flow values tend to accelerate the wave much more than it would in nature, while the converse is true if base flow reference values are used (Ponce 1983).

(3) Selection of the number of subreaches. The Muskingum equation has a constraint related to the relationship between the parameter K and the computation interval Δt. Ideally, the two should be equal, but Δt should not be less than $2KX$ to avoid negative coefficients and instabilities in the routing procedure.

$$2KX < \Delta t \le K \tag{9-23}$$

A long routing reach should be subdivided into subreaches so that the travel time through each subreach is approximately equal to the routing interval Δt. That is:

$$Number\ of\ subreaches = \frac{K}{\Delta t}$$

This assumes that factors such as channel geometry and roughness have been taken into consideration in determining the length of the routing reach and the travel time K.

d. Working R&D routing procedure. The Working R&D procedure is a storage routing technique that accommodates the nonlinear nature of floodwave movement in natural channels. The method is useful in situations where the use of a variable K (reach travel time) would assist in obtaining accurate answers. A nonlinear storage-outflow relationship indicates that a variable K is necessary. The method is also useful in situations wherein the horizontal reservoir surface assumption of the modified puls procedure is not applicable, such as normally occurs in natural channels.

(1) The working R&D procedure could be termed "Muskingum with a variable K" or "modified puls with wedge storage." For a straight line storage-discharge (weighted discharge) relation, the procedure is the same solution as the Muskingum method. For $X = 0$, the procedure is identical to Modified Puls.

(2) The basis for the procedure derives from the concept of a "working discharge," which is a hypothetical steady flow that would result in the same natural channel storage that occurs with the passage of a floodwave. Figure 9-12 illustrates this concept.

where

I = reach inflow

O = reach outflow

D = working value discharge or simply working discharge

(3) The wedge storage (WS) may be computed in the following two ways: As in the Muskingum technique where X is a weighting factor and K is reach travel time:

$$WS = KX\,(I - O) \tag{9-24}$$

or using the working discharge (D) concept:

$$WS = K\,(D - O) \tag{9-25}$$

equating and solving for O:

$$K\,(D - O) = KX\,(I - O) \tag{9-26}$$

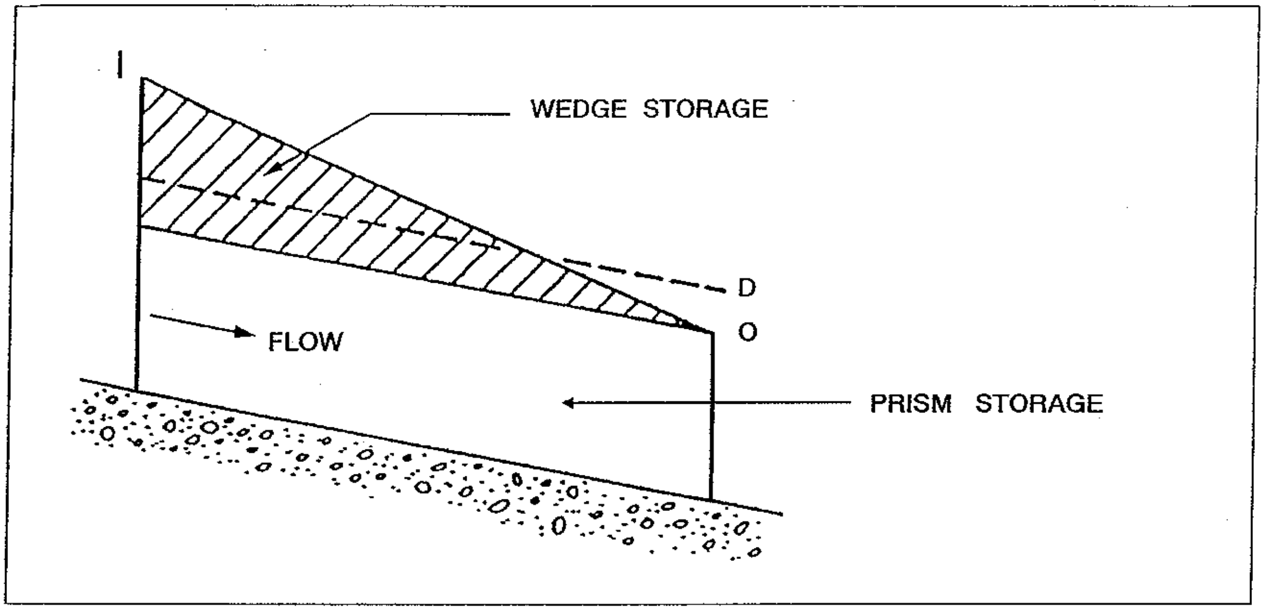

Figure 9-12. Illustration of the "working discharge" concept

or

$$O = D - \frac{X}{1-X}(I-D) \quad (9\text{-}27)$$

The continuity equation may be approximated by:

$$\frac{S_2 - S_1}{\Delta t} = 0.5\,(I_1 + I_2) - 0.5\,(O_1 + O_2) \quad (9\text{-}28)$$

where

S = storage

Δt = time increment

Substituting Equation 9-27 into 9-28 and appending the appropriate subscripts to denote beginning and end of period and performing the appropriate algebra yields:

$$0.5\Delta t(I_1 + I_2) + [S_1(1 - X) - 0.5D_1\Delta t] = [S_2(1 - X) + 0.5D_2\Delta t] \quad (9\text{-}29)$$

Let

$$R = S\,(1 - X) + 0.5D\Delta t \quad (9\text{-}30)$$

where R is termed the "working value of storage" or simply working storage and represents an index of the true natural storage. Equation 9-29 may therefore be written:

$$R_2 = R_1 + 0.5\Delta t\,(I_1 + I_2) - D_1\Delta t \quad (9\text{-}31)$$

transposing Δt results in the equation used in routing computations:

$$\frac{R_2}{\Delta t} = \frac{R_1}{\Delta t} + 0.5\,(I_1 + I_2) - D_1 \quad (9\text{-}32)$$

The form of the relationship for R (working discharge) is analogous to storage indication in the modified puls procedure. $R_2/\Delta t$ may be computed from information known at the beginning of a routing interval. The outflow at the end of the routing interval may then be determined from a rating curve of working storage versus working discharge. The cycle is then repeated stepping forward in time.

(4) The solution scheme using this concept requires development of a rating curve of working storage versus working discharge as stated above. The following column headings are helpful in developing the function when storage-outflow data are available.

1	2	3	4	5
Storage (*S*)	$\frac{S}{\Delta t}(1-X)$	*Working Discharge* (*D*)	$\frac{D}{2}$	$\frac{S}{\Delta t}(1-X) + \frac{D}{2}$

(5) Column 2 of the tabulation is obtained from column 1 by using an appropriate conversion factor and appropriate X. The conversion factor of 1 acre-ft/hour = 12.1 cfs is useful in this regard. Column 5 is the sum of columns 2 and 4. Column 3 is plotted against column 5 on cartesian coordinate paper and a curve drawn through the plotted points. This represents the working discharge-working outflow rating curve. An example curve is shown in Figure 9-13.

(6) The routing of a hydrograph can be performed as the one shown in Table 9-2. The procedure, in narrative form is:

- Conditions known at time 1: I_1, O_1, D_1, and $R_1/\Delta t$.
- At time 2, only I_2 is known, therefore:

$$\frac{R_2}{\Delta t} = \frac{R_1}{\Delta t} + 0.5\ (I_1 + I_2) - D_1$$

- Enter working storage, working discharge function, and read out D_2.
- Calculate O_2 as follows:

$$O_2 = D_2 - \frac{X}{1-X}\ (I_2 - D_2)$$

- Repeat process until finished.

e. Muskingum-Cunge channel routing. The Muskingum-Cunge channel routing technique is a nonlinear coefficient method that accounts for hydrograph diffusion based on physical channel properties and the inflowing hydrograph. The advantages of this method over other hydrologic techniques are the parameters of the model are more physically based; the method has been shown to compare well against the full unsteady flow equations over a wide range of flow situations (Ponce 1983 and Brunner 1989); and the solution is independent of the user-specified computation interval. The major limitations of the Muskingum-Cunge technique are that it cannot account for backwater effects, and the method begins to diverge from the full unsteady flow solution when very rapidly rising hydrographs are routed through flat channel sections.

(1) Development of equations.

(a) The basic formulation of the equations is derived from the continuity Equation 9-33 and the diffusion form of the momentum Equation 9-34:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q_l \qquad (9\text{-}33)$$

$$S_f = S_o - \frac{\partial Y}{\partial x} \qquad (9\text{-}34)$$

(b) By combining Equations 9-33 and 9-34 and linearizing, the following convective diffusion equation is formulated (Miller and Cunge 1975):

$$\frac{\partial Q}{\partial t} + c\frac{\partial Q}{\partial x} = \mu\frac{\partial^2 Q}{\partial x^2} + cq_L \qquad (9\text{-}35)$$

where

Q = discharge, in cubic feet per second

A = flow area, in square feet

t = time, in seconds

x = distance along the channel, in feet

Y = depth of flow, in feet

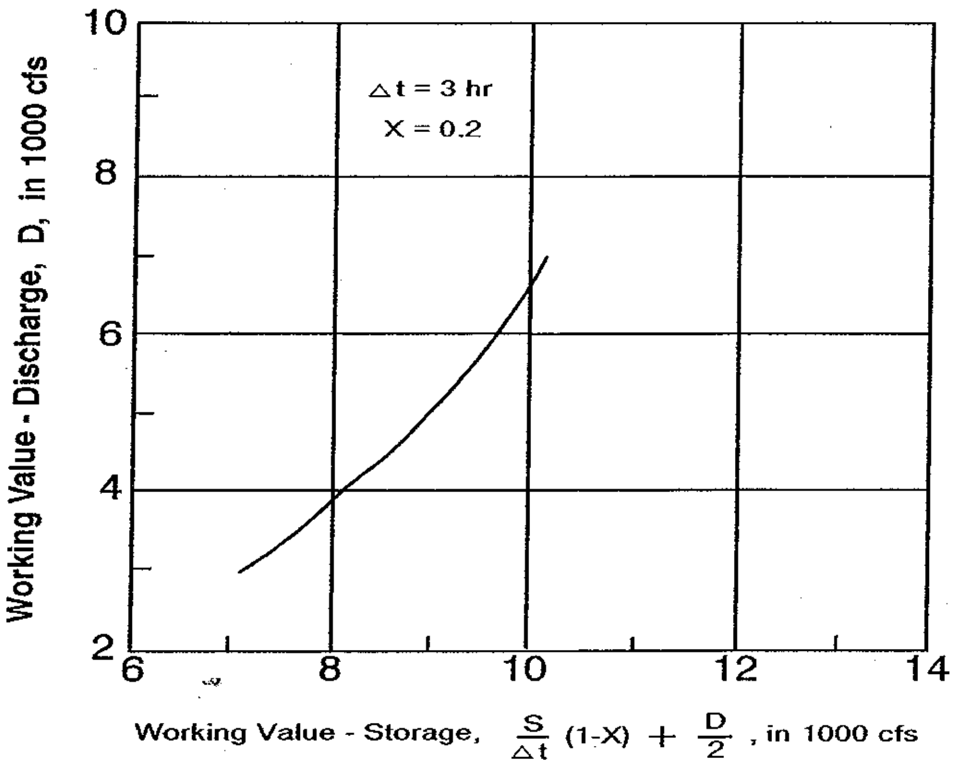

Figure 9-13. Rating curve for working R&D routing

q_L = lateral inflow per unit of channel length

S_f = friction slope

S_o = bed slope

c = the wave celerity in the x direction as defined below

The wave celerity (c) and the hydraulic diffusivity (μ) are expressed as follows:

$$c = \frac{dQ}{dA} \tag{9-36}$$

$$\mu = \frac{Q}{2BS_o} \tag{9-37}$$

where B is the top width of the water surface. The convective diffusion Equation 9-35 is the basis for the Muskingum-Cunge method.

(c) In the original Muskingum formulation, with lateral inflow, the continuity Equation 9-33) is discretized on the x-t plane (Figure 9-14) to yield:

$$Q_{j-1}^{n-1} = C_1 Q_j^n + C_2 Q_j^{n-1} + C_3 Q_{j-1}^n + C_4 Q_L \tag{9-38}$$

It is assumed that the storage in the reach is expressed as the classical Muskingum storage:

$$S = K\,[XI + (1-X)O] \tag{9-39}$$

Table 9-2
Working R&D Routing Example

Time hr	Inflow cfs	Average Inflow cfs	$\frac{K}{\Delta t} + 0.5(I_1 + I_2) - D_1$ cfs	D cfs	O cfs
	3,000		7,100	3,000	3,000
		3,130			
3	3,260		7,230	3,100	3,060
		3,445			
6	3,630		7,575	3,300	3,220
		3,825			
9	4,020		8,100	3,800	3,745
		4,250			
12	4,480		8,550	4,400	4,420

where

S = channel storage

K = cell travel time (seconds)

X = weighting factor

I = inflow

O = outflow

Therefore, the coefficients can be expressed as follows:

$$C_1 = \frac{\frac{\Delta t}{K} + 2X}{\frac{\Delta t}{K} + 2(1 - X)}$$

$$C_2 = \frac{\frac{\Delta t}{K} - 2X}{\frac{\Delta t}{K} + 2(1 - X)}$$

$$C_3 = \frac{2(1 - X) - \frac{\Delta t}{K}}{\frac{\Delta t}{K} + 2(1 - X)}$$

$$Q_L = q_L \Delta X$$

$$C_4 = \frac{2(\frac{\Delta t}{K})}{\frac{\Delta t}{K} + 2(1 - X)}$$

(d) In the Muskingum equation the amount of diffusion is based on the value of X, which varies between 0.0 and 0.5. The Muskingum X parameter is not directly related to physical channel properties. The diffusion obtained with the Muskingum technique is a function of how the equation is solved and is therefore considered numerical diffusion rather than physical. Cunge evaluated the diffusion that is produced in the Muskingum equation and analytically solved for the following diffusion coefficient:

$$\mu_n = c\,\Delta x \left(\frac{1}{2} - X \right) \tag{9-40}$$

In the Muskingum-Cunge formulation, the amount of diffusion is controlled by forcing the numerical diffusion to match the physical diffusion of the convective diffusion Equation 9-35. This is accomplished by setting Equations 9-37 and 9-40 equal to each other. The Muskingum-Cunge equation is therefore considered an approximation of the convective diffusion Equation 9-35. As a result, the parameters K and X are expressed as follows (Cunge 1969 and Ponce and Yevjevich 1978):

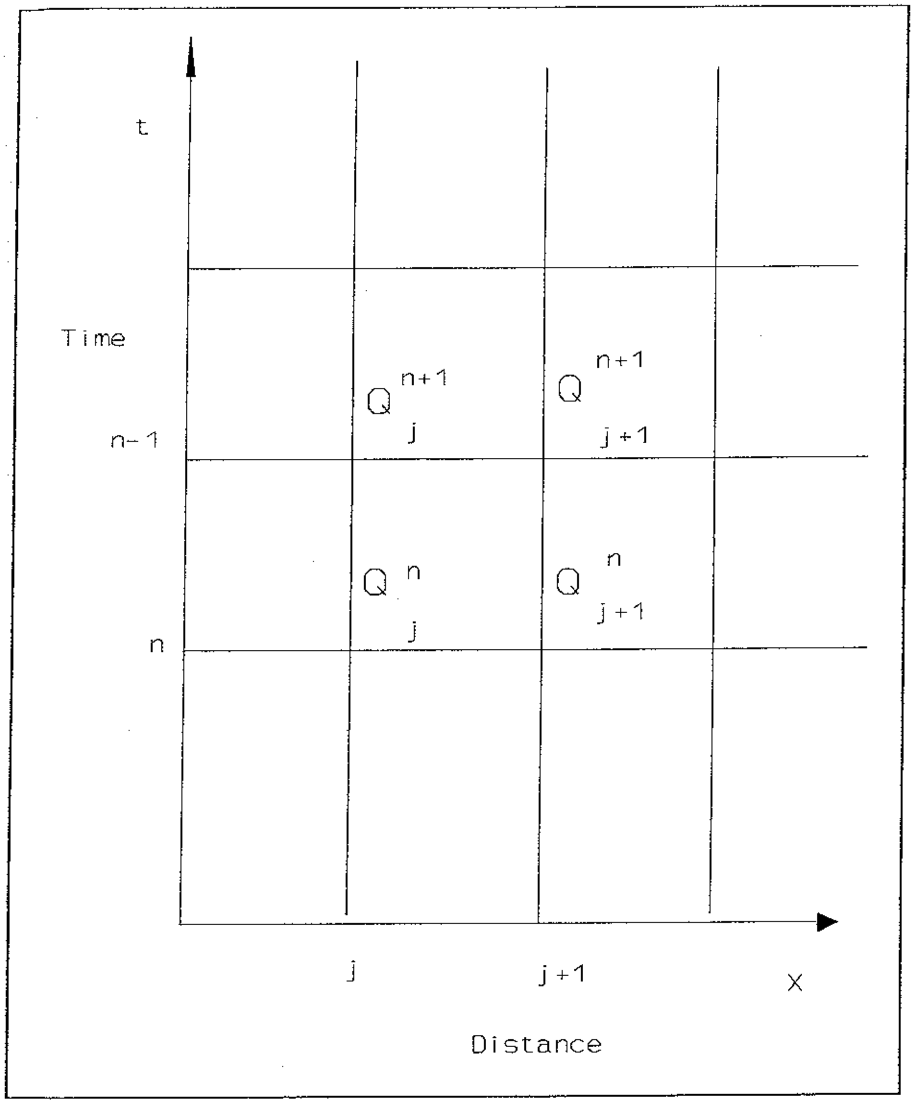

Figure 9-14. Discretization of the continuity equation on x-t plane

$$K = \frac{\Delta X}{c} \tag{9-41}$$

$$X = \frac{1}{2}\left(1 - \frac{Q}{BS_o c\Delta x}\right) \tag{9-42}$$

(2) Solution of the equations.

(a) The method is nonlinear in that the flow hydraulics (Q, B, c), and therefore the routing coefficients (C_1, C_2, C_3, and C_4) are recalculated for every Δx distance step and Δt time step. An iterative four-point averaging scheme is used to solve for c, B, and Q. This process has been described in detail by Ponce (1986).

(b) Values for Δt and Δx are chosen for accuracy and stability. First, Δt should be evaluated by looking at the following three criteria and selecting the smallest value: (1) the user-defined computation interval, (2) the time of rise of the inflow hydrograph divided by 20 ($^{Tr}/_{20}$), and (3) the travel time through the channel reach. Once Δt is chosen, Δx is defined as follows:

$$\Delta x = c\Delta t \tag{9-43}$$

but Δx must also meet the following criteria to preserve consistency in the method (Ponce 1983):

$$\Delta x < \frac{1}{2}\left(c\Delta t + \frac{Q_o}{BS_o c}\right) \quad (9\text{-}44)$$

where Q_o is the reference flow and Q_B is the baseflow taken from the inflow hydrograph as:

$$Q_o = Q_B + 0.50\ (Q_{peak} - Q_B)$$

(3) Data requirements.

(a) Data for the Muskingum-Cunge method consist of the following:

- Representative channel cross section.
- Reach length, L.
- Manning roughness coefficients, n (for main channel and overbanks).
- Friction slope (S_f) or channel bed slope (S_o).

(b) The method can be used with a simple cross section (i.e., trapezoid, rectangle, square, triangle, or circular pipe) or a more detailed cross section (i.e., cross sections with a left overbank, main channel, and a right overbank). The cross section is assumed to be representative of the entire routing reach. If this assumption is not adequate, the routing reach should be broken up into smaller subreaches with representative cross sections for each. Reach lengths are measured directly from topographic maps. Roughness coefficients (Manning's n) must be estimated for main channels as well as overbank areas. If information is available to estimate an approximate energy grade line slope (friction slope, S_f), that slope should be used instead of the bed slope. If no information is available to estimate the slope of the energy grade line, the channel bed slope should be used.

(4) Advantages and limitations. The Muskingum-Cunge routing technique is considered to be a nonlinear coefficient method that accounts for hydrograph diffusion based on physical channel properties and the inflowing hydrograph. The advantages of this method over other hydrologic techniques are: the parameters of the model are physically based, and therefore this method will make for a good ungauged routing technique; several studies have shown that the method compares very well with the full unsteady flow equations over a wide range of flow conditions (Ponce 1983 and Brunner 1989); and the solution is independent of the user-specified computation interval. The major limitations of the Muskingum-Cunge technique are that the method can not account for backwater effects, and the method begins to diverge from the complete unsteady flow solution when very rapidly rising hydrographs (i.e., less than 2 hr) are routed through flat channel sections (i.e., channel slopes less than 1 ft/mile). For hydrographs with longer rise times (T_r), the method can be used for channel reaches with slopes less than 1 ft/mile.

9-4. Applicability of Routing Techniques

a. Selecting the appropriate routing method. With such a wide range of hydraulic and hydrologic routing techniques, selecting the appropriate routing method for each specific problem is not clearly defined. However, certain thought processes and some general guidelines can be used to narrow the choices, and ultimately the selection of an appropriate method can be made.

b. Hydrologic routing method. Typically, in rainfall-runoff analyses, hydrologic routing procedures are utilized on a reach-by-reach basis from upstream to downstream. In general, the main goal of the rainfall-runoff study is to calculate discharge hydrographs at several locations in the watershed. In the absence of significant backwater effects, the hydrologic routing models offer the advantages of simplicity, ease of use, and computational efficiency. Also, the accuracy of hydrologic methods in calculating discharge hydrographs is normally well within the range of acceptable values. It should be remembered, however, that insignificant backwater effects alone do not always justify the use of a hydrologic method. There are many other factors that must be considered when deciding if a hydrologic model will be appropriate, or if it is necessary to use a more detailed hydraulic model.

c. Hydraulic routing method. The full unsteady flow equations have the capability to simulate the widest range of flow situations and channel characteristics. Hydraulic models, in general, are more physically based since they only have one parameter (the roughness coefficient) to estimate or calibrate. Roughness coefficients can be estimated with some degree of accuracy from inspection of the waterway, which makes the hydraulic methods more applicable to ungauged situations.

d. Evaluating the routing method. There are several factors that should be considered when evaluating which routing method is the most appropriate for a given