Similarly, the momentum equation can be modified by multiplying by Δx_i to obtain

$$\frac{\Delta x_i}{2\Delta t_j}\left(Q_i^{j+1}+Q_{i+1}^{j+1}-Q_i^j-Q_{i+1}^j\right)$$

$$+\theta\left\{\left(\frac{\beta Q^2}{A}\right)_{i+1}^{j+1}-\left(\frac{\beta Q^2}{A}\right)_i^{j+1}+g\bar{A}_i^{j+1}\left[h_{i+1}^{j+1}-h_i^{j+1}+\left(\bar{S}_f\right)_i^{j+1}\Delta x_i+\left(\bar{S}_e\right)_i^{j+1}\Delta x_i\right]-\left(\overline{\beta q v_x}\right)_i^{j+1}\Delta x_i\right\}$$

$$+(1-\theta)\left\{\left(\frac{\beta Q^2}{A}\right)_{i+1}^{j}-\left(\frac{\beta Q^2}{A}\right)_i^{j}+g\bar{A}_i^{j}\left[h_{i+1}^{j}-h_i^{j}+\left(\bar{S}_f\right)_i^{j}\Delta x_i+\left(\bar{S}_e\right)_i^{j}\Delta x_i\right]-\left(\overline{\beta q v_x}\right)_i^{j}\Delta x_i\right\}=0$$

(9.7.10)

where the average values (marked with an overbar) over a reach are defined as

$$\bar{\beta}_i=\frac{\beta_i+\beta_{i+1}}{2} \tag{9.7.11}$$

$$\bar{A}_i=\frac{A_i+A_{i+1}}{2} \tag{9.7.12}$$

$$\bar{B}_i=\frac{B_i+B_{i+1}}{2} \tag{9.7.13}$$

$$\bar{Q}_i=\frac{Q_i+Q_{i+1}}{2} \tag{9.7.14}$$

Also,

$$\bar{R}_i=\bar{A}_i/\bar{B}_i \tag{9.7.15}$$

for use in Manning's equation. Manning's equation may be solved for S_f and written in the form shown below, where the term $|Q|Q$ has magnitude Q^2 and sign positive or negative depending on whether the flow is downstream or upstream, respectively:

$$\left(\bar{S}_f\right)_i=\frac{\bar{n}_i^2|\bar{Q}_i|\bar{Q}_i}{2.208\bar{A}_i^2\bar{R}_i^{4/3}} \tag{9.7.16}$$

The minor headlosses arising from contraction and expansion of the channel are proportional to the difference between the squares of the downstream and upstream velocities, with a contraction/expansion loss coefficient K_e:

$$\left(\bar{S}_e\right)_i=\frac{(K_e)_i}{2g\Delta x_i}\left[\left(\frac{Q}{A}\right)_{i+1}^2-\left(\frac{Q}{A}\right)_i^2\right] \tag{9.7.17}$$

The terms having superscript j in equations (9.7.9) and (9.7.10) are known either from initial conditions or from a solution of the Saint–Venant equations for a previous time line. The terms g, Δx_i, β_i, K_e, C_w, and V_w are known and must be specified independently of the solution. The unknown terms are Q_i^{j+1}, Q_{i+1}^{j+1}, h_{i+1}^{j+1}, A_i^{j+1}, $A_{i+1}^{j+1}B_i^{j+1}$, and B_{i+1}^{j+1}. However, all the terms can be expressed as functions of the unknowns Q_i^{j+1}, Q_{i+1}^{j+1}, h_i^{j+1}, and h_{i+1}^{j+1}, so there are actually four unknowns. The unknowns are raised to powers other than unity, so equations (9.7.9) and (9.7.10) are nonlinear equations.

The continuity and momentum equations are considered at each of the N-1 rectangular grids shown in Figure 9.7.1, between the upstream boundary at i = 1 and the downstream boundary at

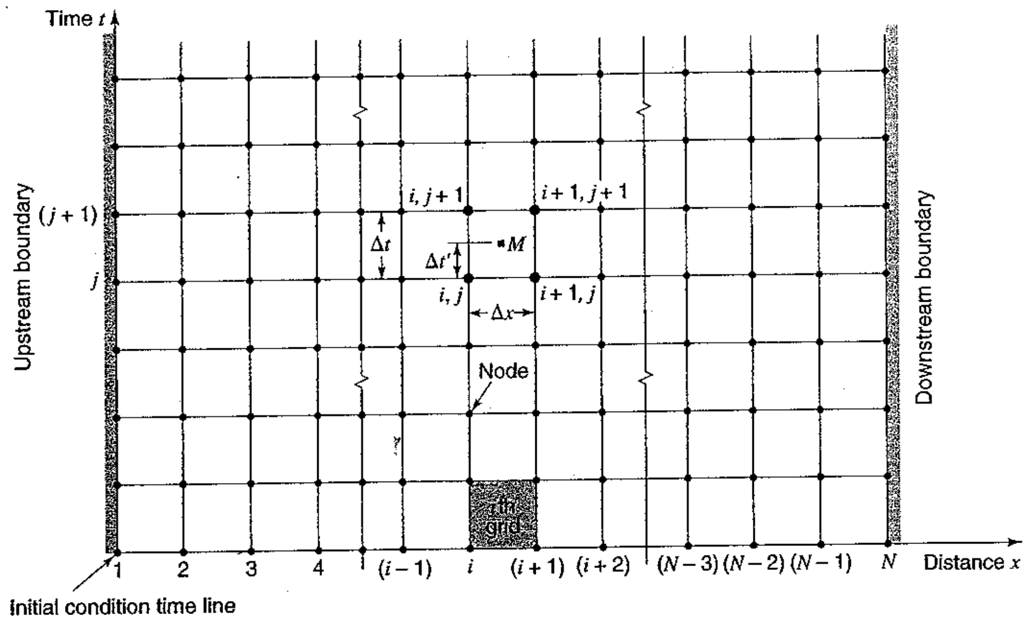


Figure 9.7.1 The x-t solution plane. The finite-difference forms of the Saint–Venant equations are solved at a discrete number of points (values of the independent variables x and t) arranged to form the rectangular grid shown. Lines parallel to the time axis represent locations along the channel, and those parallel to the distance axis represent times (from Fread (1974)).

$i = N$. This yields $2N$-2 equations. There are two unknowns at each of the N grid points (Q and h), so there are $2N$ unknowns in all. The two additional equations required to complete the solution are supplied by the upstream and downstream boundary conditions. The upstream boundary condition is usually specified as a known inflow hydrograph, while the downstream boundary condition can be specified as a known stage hydrograph, a known discharge hydrograph, or a known relationship between stage and discharge, such as a rating curve. The U.S. National Weather Service FLDWAV model (hsp.nws.noaa.gov/oh/hrl/rvmech) uses the above to describe implicit dynamic wave model formulation.

PROBLEMS

9.1.1 The storage-outflow characteristics for a reservoir are given below. Determine the storage-outflow function $2S/\Delta t + Q$ versus Q for each of the tabulated values using $\Delta t = 1.0$ hr. Plot a graph of the storage-outflow function.

Storage (106 m^3)	70	80	85	100	115
Outflow (m^3)	0	50	150	350	700

9.2.1 Route the inflow hydrograph given below through the reservoir with the storage-outflow characteristics given in problem 3.6.1 using the level pool method. Assume the reservoir has an initial storage of 70×106 m^3.

Time (h)	0	1	2	3	4	5	6	7	8
Inflow (m^3/s)	0	40	60	150	200	300	250	200	180

Time (h)	9	10	11	12	13	14	15	16
Inflow(m^3/s)	220	320	400	280	190	150	50	0

9.2.2 Rework problem 9.2.1 assuming the reservoir storage is initially 80×10^3 m^3.

9.2.3 Write a computer program to solve problems 9.2.1 and 9.2.2.

9.2.4 Rework example 9.1.1 using a 1.5-acre detention basin.

9.2.5 Rework example 9.1.1 using a triangular inflow hydrograph that increases linearly from zero to a peak of 90 cfs at 120 min and then decreases linearly to a zero discharge at 240 min. Use a 30-min routing interval.

9.2.6 Rework example 9.2.2 using $\Delta t = 2$ hrs.

9.2.7 Rework example 9.2.2 assuming $X = 0.3$ hrs.

9.3.1 Rework example 9.2.2 assuming $K = 1.4$ hr.

9.3.2 Calculate the Muskingum routing K and number of routing steps for a 1.25-mi long channel. The average cross-section dimensions for the channel are a base width of 25 ft and an average depth of 2.0 ft. Assume the channel is rectangular and has Manning's n 0.04 and a slope of 0.009 ft/ft.

9.3.3 Route the following upstream inflow hydrograph through a downstream flood control channel reach using the Muskingum method. The channel reach has a $K = 2.5$ hr and $X = 0.2$. Use a routing interval of 1 hr.

Time (h)	1	2	3	4	5	6	7
Inflow (cfs)	90	140	208	320	440	550	640
Time (h)	8	9	10	11	12	13	14
Inflow (cfs)	680	690	630	570	470	390	
Time (h)	15	16	17	18	19	20	
Inflow (cfs)	330	250	180	130	100	90	

9.3.4 Use the U.S. Army Corps of Engineers HEC-1 computer program to solve Problem 9.3.3.

REFERENCES

Alley, W. M., and P. E. Smith, Distributed Routing Rainfall-Runoff Model, Open File Report 82–344, U.S. Geological Survey, Reston, VA, 1987.

Bradley, J., *Hydraulics of Bridge Water Way*, Hydraulic Design Sevies No. 1, Federal Highway Administration. U.S. Department of Transportation, Washington, DC, 1978.

Chow, V. T., *Open Channel Hydraulics*, McGraw-Hill, New York, 1959.

Chow, V. T. (editor-in-chief), *Handbook of Applied Hydrology*, McGraw-Hill, New York, 1964.

Chow, V. T., D. R. Maidment, and L. W. Mays, *Applied Hydrology*, McGraw-Hill, New York, 1988.

Cudworth, A. G., Jr., *Flood Hydrology Manual*, U. S. Department of the Interior, Bureau of Reclamation, Denver, CO, 1989.

Cunge, J. A., "On the Subject of a Flood Propagation Method (Muskingum Method)," *Journal of Hydraulics Research*, International Association of Hydraulic Research, vol. 7, no, 2, pp. 205–230, 1969.

Fread, D. L., "Discussion of 'Implicit Flood Routing in Natural Channels,' by M. Amein and C. S. Fang," *Journal of the Hydraulics Division, ASCE*, vol. 97, no HY.7, pp. 1156–1159, 1971.

Fread, D. L., *Numerical Properties of Implicit Form-Point Finite Difference Equation of Unsteady Flow*, NOAA Technical Memorandum NWS HYDRO 18, National Weather Service, NOAA, U.S. Dept. of Commerce, Silver Spring, MD, 1974.

Fread, D. L., "Theoretical Development of Implicit Dynamic Routing Model," Dynamic Routing Service at Lower Mississippi River Forecast Center, Slidell, Louisiana, National Weather Service, NOAA, Silver Spring, MD, 1976.

Henderson, F. M., *Open Channel Flow*, Macmillan, New York, 1966.

Hewlett, J. D., *Principles of Forest Hydrology*, University of Georgia Press, Athens, GA, 1982.

Lighthill, M. J., and G. B. Whitham, "On Kinematic Waves, I: Flood Movement in Long Rivers," *Proc. Roy. Soc. London* A, vol. 229, no. 1178, pp. 281–316, 1955.

Maidment, D. R. (editor-in-chief), *Handbook of Hydrology*, McGraw-Hill, New York, 1993.

Masch, F. D., *Hydrology*, Hydraulic Engineering Circular No. 19, FHWA-10-84-15, Federal Highway Administration, U.S. Department of the Interior, McLean, VA, 1984.

Mays, L. W., and Y. K. Tung, *Hydrosystems Engineering and Management*, McGraw-Hill, New York, 1992.

McCuen, R. H., *Hydrologic Analysis and Design*, Prentice-Hall, Englewood Cliffs, NJ, 1989.

Morris, E. M., and D. A. Woolhiser, 1980, "Unsteady One-Dimensional Flow over a Plane: Partial Equilibrium and Recession Hydrographs," *Water Resources Research* 16(2): 355–360.

Mosley, M. P., and A. I. McKerchar, "Streamflow," in *Handbook of Hydrology* (edited by D. R. Maidment), McGraw-Hill, New York, 1993.

Ponce, V. M., *Engineering Hydrology: Principles and Practices*, Prentice-Hall, Englewood Cliffs, NJ, 1989.

Strahler, A. N., "Quantitative Geomorphology of Drainage Basins and Channel Networks," section 4–II in *Handbook of Applied Hydrology*, edited by V. T. Chow, pp. 4–39, 4–76, McGraw-Hill, New York, 1964.

Strelkoff, T., "Numerical Solution of Saint–Venant Equation," *Journal of the Hydraulics Division, ASCE*, vol. 96, no. HY1, pp. 223–252, 1970.

Strelkoff, T., The One-Dimensional Equations of Open-Channel Flow, *Journal of the Hydraulics Division*, American Society of Civil Engineers, vol. 95, no. Hy3, pp. 861–874, 1969.

U.S. Army Corps of Engineers, "Routing of Floods Through River Channels," *Engineer Manual*, 1110–2–1408, Washington, DC, 1960.

U.S. Army Corps of Engineers, Hydrologic Engineering Center, *HEC-1, Flood Hydrograph Package, User's Manual*, Davis, CA, 1990.

U.S. Department of Agriculture, Soil Conservation Service, "A Method for Estimating Volume and Rate of Runoff in Small Watersheds," Tech. Paper 149, Washington, DC, 1973.

U.S. Department of Agriculture, Soil Conservation Service, "Urban Hydrology for Small Watersheds," Tech. Release no. 55, Washington, DC, 1986.

U.S. Environmental Data Services, *Climate Atlas of the U.S.*, U.S. Government Printing Office, Washington, DC, pp. 43–44, 1968.

U.S. National Research Council, Committee on Opportunities in the Hydrologic Sciences, Water Science and Technology Board, *Opportunities in the Hydrologic Sciences*, National Academy Press, Washington, DC, 1991.

Viessman, W., Jr., and G. L. Lewis, *Introduction to Hydrology*, fourth edition, Harper and Row, New York, 1996.

Woolhiser, D. A., and J. A. Liggett, Unsteady, One-Dimensional Flow Over a Plane—the Rising Hydrograph, *Water Resources Research*, vol. 3(3), pp. 753–771, 1967.

Woolhiser, D. A., R. E. Smith, and D. C. Goodrich, *KINEROS, A Kinematic Runoff and Erosion Model: Documentation and User Manual*, U. S. Department of Agricultural Research Service, ARS-77, Tucson, AZ, 1990.

HAND OUT 19: Overview of hydraulic routing (Chapter 6 of our syllabus).

Overview of hydraulic routing

1) General equations

Mass: $$\boxed{\frac{\partial Q}{\partial x} + \frac{\partial \Omega}{\partial t} = q}$$

$\Omega \equiv A$ area

Momentum:

$$\boxed{\frac{\partial Q}{\partial t} + \frac{\partial}{\partial x}\left(\beta \frac{Q^2}{\Omega}\right) + g\Omega\left(\frac{\partial y}{\partial x} + S_f + S_e\right) - \beta q v_x = 0}$$

2) Approximations

a) Kinematic wave:

KW: $$\frac{\partial Q}{\partial x} + \frac{\partial \Omega}{\partial t} = q$$

$$S_o = S_f$$

b) "Diffusive" wave:

$$\frac{\partial Q}{\partial x} + \frac{\partial \Omega}{\partial t} = q$$

$$g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0$$

3) Solutions for the KW

a) HEC-1, version 2)

$$\frac{\partial \Omega}{\partial t} = \frac{\Omega^{j+1}_{\text{[redacted]}\,i+1} - \Omega^{j}_{i+1}}{\Delta t}$$

$$\frac{\partial \Omega}{\partial x} = \frac{\Omega^{j}_{i+1} - \Omega^{j}_{i}}{\Delta x}$$

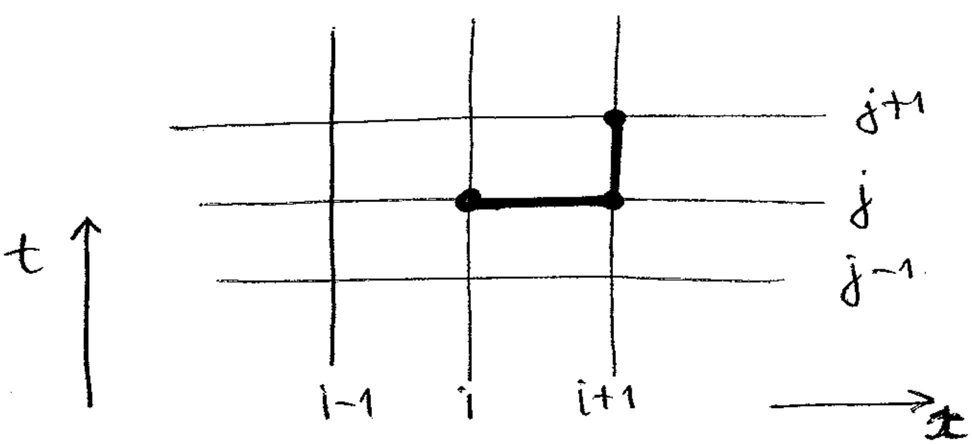
$$\Omega = \frac{\Omega^{j}_{i+1} + \Omega^{j}_{i}}{2}$$

$$q = \frac{q^{j+1}_{i+1} + q^{j}_{i+1}}{2}$$

B-1

$$\Rightarrow \Omega^{j+1}_{i+1} = \Omega^{j}_{i+1} - aB\left(\frac{\Delta t}{\Delta x}\right)\left[\frac{\Omega^{j}_{i+1} + \Omega^{j}_{i}}{2}\right] \times$$

$$\times \left(\Omega^{j}_{i+1} - \Omega^{j}_{i}\right) + \left(q^{j+1}_{i+1} + q^{j}_{i+1}\right)\frac{\Delta t}{2}$$



$$Q_{i+1}^{j+1} = a\left(\Omega_{i+1}^{j+1}\right)^{B}$$

b) HEC-1, Version b)

$$\frac{\partial Q}{\partial x} = \frac{Q_{i+1}^{j+1} - Q_i^{j+1}}{\Delta x}$$

$$\frac{\partial \Omega}{\partial t} = \frac{\Omega_i^{j+1} - A_i^{j}}{\Delta t}$$

$$Q_{i+1}^{j+1} = Q_i^{j+1} + q\,\Delta x - \frac{\Delta x}{\Delta t}\left(\Omega_i^{j+1} - \Omega_i^{j}\right)$$

HAND OUT 20: Overview of hydrologic and hydraulic routing (Chapter 6 of our syllabus). Source: U.S. Corps of Engineers.

Chapter 9
Streamflow and Reservoir Routing

9-1. General

a. Routing is a process used to predict the temporal and spatial variations of a flood hydrograph as it moves through a river reach or reservoir. The effects of storage and flow resistance within a river reach are reflected by changes in hydrograph shape and timing as the floodwave moves from upstream to downstream. Figure 9-1 shows the major changes that occur to a discharge hydrograph as a floodwave moves downstream.

b. In general, routing techniques may be classified into two categories: hydraulic routing, and hydrologic routing. Hydraulic routing techniques are based on the solution of the partial differential equations of unsteady open channel flow. These equations are often referred to as the St. Venant equations or the dynamic wave equations. Hydrologic routing employs the continuity equation and an analytical or an empirical relationship between storage within the reach and discharge at the outlet.

c. Flood forecasting, reservoir and channel design, floodplain studies, and watershed simulations generally utilize some form of routing. Typically, in watershed simulation studies, hydrologic routing is utilized on a reach-by-reach basis from upstream to downstream. For example, it is often necessary to obtain a discharge hydrograph at a point downstream from a location where a hydrograph has been observed or computed. For such purposes, the upstream hydrograph is routed through the reach with a hydrologic routing technique that predicts changes in hydrograph shape and timing. Local flows are then added at the downstream location to obtain the total flow hydrograph. This type of approach is adequate as long as there are no significant backwater effects or

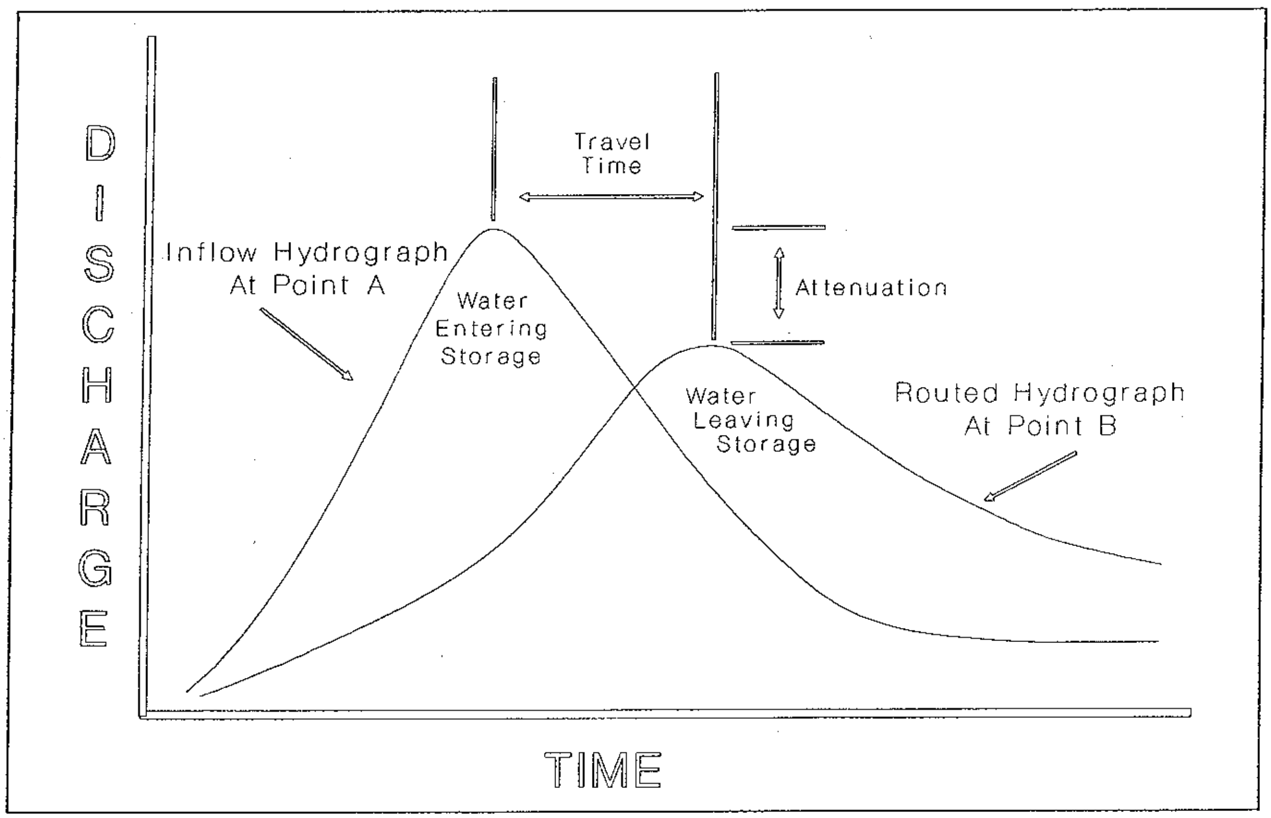


Figure 9-1. Discharge hydrograph routing effects

discontinuities in the water surface because of jumps or bores. When there are downstream controls that will have an effect on the routing process through an upstream reach, the channel configuration should be treated as one continuous system. This can only be accomplished with a hydraulic routing technique that can incorporate backwater effects as well as internal boundary conditions, such as those associated with culverts, bridges, and weirs.

d. This chapter describes several different hydraulic and hydrologic routing techniques. Assumptions, limitations, and data requirements are discussed for each. The basis for selection of a particular routing technique is reviewed, and general calibration methodologies are presented. This chapter is limited to discussions on 1-D flow routing techniques in the context of flood-runoff analysis. The focus of this chapter is on discharge (flow) rather than stage (water surface elevation). Detailed presentation of routing techniques and applications focused on stage calculations can be found in EM 1110-2-1416.

9-2. Hydraulic Routing Techniques

a. The equations of motion. The equations that describe 1-D unsteady flow in open channels, the Saint Venant equations, consist of the continuity equation, Equation 9-1, and the momentum equation, Equation 9-2. The solution of these equations defines the propagation of a floodwave with respect to distance along the channel and time.

$$A\frac{\partial V}{\partial x} + VB\frac{\partial y}{\partial x} + B\frac{\partial y}{\partial t} = q \qquad (9\text{-}1)$$

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{V}{g}\frac{\partial V}{\partial x} - \frac{1}{g}\frac{\partial V}{\partial t} \qquad (9\text{-}2)$$

where

A = cross-sectional flow area

V = average velocity of water

x = distance along channel

B = water surface width

y = depth of water

t = time

q = lateral inflow per unit length of channel

S_f = friction slope

S_o = channel bed slope

g = gravitational acceleration

Solved together with the proper boundary conditions, Equations 9-1 and 9-2 are the complete dynamic wave equations. The meaning of the various terms in the dynamic wave equations are as follows (Henderson 1966):

(1) Continuity equation.

$A\frac{\partial V}{\partial x}$ = *prism storage*

$VB\frac{\partial y}{\partial x}$ = *wedge storage*

$B\frac{\partial y}{\partial t}$ = *rate of rise*

q = *lateral inflow per unit length*

(2) Momentum equation.

S_f = *friction slope (frictional forces)*

S_o = *bed slope (gravitational effects)*

$\frac{\partial y}{\partial x}$ = *pressure differential*

$\frac{V}{g}\frac{\partial V}{\partial x}$ = *convective acceleration*

$\frac{1}{g}\frac{\partial V}{\partial t}$ = *local acceleration*

(3) Dynamic wave equations. The dynamic wave equations are considered to be the most accurate and comprehensive solution to 1-D unsteady flow problems in open channels. Nonetheless, these equations are based on specific assumptions, and therefore have limitations. The assumptions used in deriving the dynamic wave equations are as follows:

(a) Velocity is constant and the water surface is horizontal *across* any channel section.

(b) All flows are gradually varied with hydrostatic pressure prevailing at all points in the flow, such that vertical accelerations can be neglected.

(c) No lateral secondary circulation occurs.

(d) Channel boundaries are treated as fixed; therefore, no erosion or deposition occurs.

(e) Water is of uniform density, and resistance to flow can be described by empirical formulas, such as Manning's and Chezy's equation.

(f) The dynamic wave equations can be applied to a wide range of 1-D flow problems; such as, dam break floodwave routing, forecasting water surface elevations and velocities in a river system during a flood, evaluating flow conditions due to tidal fluctuations, and routing flows through irrigation and canal systems. Solution of the full equations is normally accomplished with an explicit or implicit finite difference technique. The equations are solved for incremental times (Δt) and incremental distances (Δx) along the waterway.

b. Approximations of the full equations. Depending on the relative importance of the various terms of the momentum Equation 9-2, the equation can be simplified for various applications. Approximations to the full dynamic wave equations are created by combining the continuity equation with various simplifications of the momentum equation. The most common approximations of the momentum equation are:

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{V}{g}\frac{\partial V}{\partial x} - \frac{1}{g}\frac{\partial V}{\partial t} \qquad (9\text{-}3)$$

Steady Uniform Flow
Kinematic Wave Approx.

Steady Nonuniform Flow
Diffusion Wave Approximation

Steady Nonuniform Flow
Quasi-Steady Dynamic Wave Approximation

Unsteady Nonuniform Flow
Full Dynamic Wave Equation

The use of approximations to the full equations for unsteady flow can be justified when specific terms in the momentum equation are small in comparison to the bed slope. This is best illustrated by an example taken from Henderson's book *Open Channel Flow* (1966). Henderson computed values for each of the terms on the right-hand side of the momentum equation for a steep alluvial stream:

Term:	S_o	$\frac{\partial y}{\partial x}$	$\frac{V}{g}\frac{\partial V}{\partial x}$	$\frac{1}{g}\frac{\partial V}{\partial t}$
Magnitude (ft/mi):	26	.5	.12-.25	.05

These figures relate to a very fast rising hydrograph in which the flow increased from 10,000 to 150,000 cfs and decreased again to 10,000 cfs within 24 hr. Even in this case, where changes in depth and velocity with respect to distance and time are relatively large, the last three terms are still small in comparison to the bed slope. For this type of flow situation (steep stream), an approximation of the full equations would be appropriate. For flatter slopes, the last three terms become increasingly more important.

(1) Kinematic wave approximation. Kinematic flow occurs when gravitational and frictional forces achieve a balance. In reality, a true balance between gravitational and frictional forces never occurs. However, there are flow situations in which gravitational and frictional forces approach an equilibrium. For such conditions, changes in depth and velocity with respect to time and distance are small in magnitude when compared to the bed slope of the channel. Therefore, the terms to the right of the bed slope in Equation 9-3 are assumed to be negligible. This assumption reduces the momentum equation to the following:

$$S_f = S_o \qquad (9\text{-}4)$$

Equation 9-4 essentially states that the momentum of the flow can be approximated with a uniform flow assumption as described by Manning's or Chezy's equation. Manning's equation can be written in the following form:

$$Q = \alpha A^m \qquad (9\text{-}5)$$

where α and m are related to flow geometry and surface roughness

Since the momentum equation has been reduced to a simple functional relationship between area and discharge, the movement of a floodwave is described solely by the continuity equation, written in the following form:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = q \tag{9-6}$$

Then by combining Equations 9-5 and 9-6, the governing kinematic wave equation is obtained as:

$$\frac{\partial A}{\partial t} + \alpha m A^{(m-1)} \frac{\partial A}{\partial x} = q \tag{9-7}$$

Because of the steady uniform flow assumptions, the kinematic wave equations do not allow for hydrograph diffusion, just simple translation of the hydrograph in time. The kinematic wave equations are usually solved by explicit or implicit finite difference techniques. Any attenuation of the peak flow that is computed using the kinematic wave equations is due to errors inherent in the finite difference solution scheme.

(a) The application of the kinematic wave equation is limited to flow conditions that do not demonstrate appreciable hydrograph attenuation. In general, the kinematic wave approximation works best when applied to steep (10 ft/mile or greater), well defined channels, where the floodwave is gradually varied.

(b) The kinematic wave approach is often applied in urban areas because the routing reaches are generally short and well defined (i.e., circular pipes, concrete lined channels, etc.).

(c) The kinematic wave equations cannot handle backwater effects since, with a kinematic model flow, disturbances can only propagate in the downstream direction. All of the terms in the momentum equation that are used to describe the propagation of the floodwave upstream (backwater effects) have been excluded.

(2) Diffusion wave approximation. Another common approximation of the full dynamic wave equations is the diffusion wave analogy. The diffusion wave model utilizes the continuity Equation 9-1 and the following simplified form of the momentum equation:

$$S_f = S_o - \frac{\partial y}{\partial x} \tag{9-8}$$

The diffusion wave model is a significant improvement over the kinematic wave model because of the inclusion of the pressure differential term in Equation 9-8. This term allows the diffusion model to describe the attenuation (diffusion effect) of the floodwave. It also allows the specification of a boundary condition at the downstream extremity of the routing reach to account for backwater effects. It does not use the inertial terms (last two terms) from Equation 9-2 and, therefore, is limited to slow to moderately rising floodwaves (Fread 1982). However, most natural floodwaves can be described with the diffusion form of the equations.

(3) Quasi-steady dynamic wave approximation. The third simplification of the full dynamic wave equations is the quasi-steady dynamic wave approximation. This model utilizes the continuity equation, Equation 9-1, and the following simplification of the momentum equation:

$$S_f = S_o - \frac{\partial y}{\partial x} - \frac{V}{g}\frac{\partial V}{\partial x} \tag{9-9}$$

In general, this simplification of the dynamic wave equations is not used in flood routing. This form of the momentum equation is more commonly used in steady flow-water surface profile computations. In the case of flood routing, the last two terms on the momentum equation are often opposite in sign and tend to counteract each other (Fread 1982). By including the convective acceleration term and not the local acceleration term, an error is introduced. This error is of greater magnitude than the error that results when both terms are excluded, as in the diffusion wave model. For steady flow-water surface profiles, the last term of the momentum equation (changes in velocity with respect to time) is assumed to be zero. However, changes in velocity with respect to distance are still very important in the calculation of steady flow-water surface profiles.

c. Data requirements. In general, the data requirements of the various hydraulic routing techniques are virtually the same. However, the amount of detail that is required for each type of data will vary depending upon the routing technique being used and the situation it is being applied to. The basic data requirements for hydraulic routing techniques are the following:

(1) Flow data (hydrographs).

(2) Channel cross sections and reach lengths.

(3) Roughness coefficients.

(4) Initial and boundary conditions.

(a) Flow data consist of discharge hydrographs from upstream locations as well as lateral inflow and tributary flow for all points along the stream.

(b) Channel cross sections are typically surveyed sections that are perpendicular to the flow lines. Key issues in selecting cross sections are the accuracy of the surveyed data and the spacing of the sections along the stream. If the routing procedure is utilized to predict stages, then the accuracy of the cross-sectional dimensions will have a direct effect on the prediction of the stage. If the cross sections are used only to route discharge hydrographs, then it is only important to ensure that the cross section is an adequate representation of the discharge versus flow area of the section. Simplified cross-sectional shapes, such as 8-point cross sections or trapezoids and rectangles, are often used to fit the discharge versus flow area of a more detailed section. Cross-sectional spacing affects the level of detail of the results as well as the accuracy of the numerical solution to the routing equations. Detailed discussions on cross-sectional spacing can be found in the reference by the Hydrologic Engineering Center (HEC) (USACE 1986).

(c) Roughness coefficients for hydraulic routing models are typically in the form of Manning's n values. Manning's coefficients have a direct impact on the travel time and amount of diffusion that will occur when routing a flood hydrograph through a channel reach. Roughness coefficients will also have a direct impact on predicted stages.

(d) All hydraulic models require that initial and boundary conditions be established before the routing can commence. Initial conditions are simply stated as the conditions at all points in the stream at the beginning of the simulation. Initial conditions are established by specifying a base flow within the channel at the start of the simulation. Channel depths and velocities can be calculated through steady-state backwater computations or a normal depth equation (e.g., Manning's equation). Boundary conditions are known relationships between discharge and time and/or discharge and stage. Hydraulic routing computations require the specification of upstream, downstream, and internal boundary conditions to solve the equations. The upstream boundary condition is the discharge (or stage) versus time relationship of the hydrograph to be routed through the reach. Downstream boundary conditions are usually established with a steady-state rating curve (discharge versus depth relationship) or through normal depth calculations (Manning's equation). Internal boundary conditions consist of lateral inflow or tributary flow hydrographs, as well as depth versus discharge relationships for hydraulic structures within the river reach.

9-3. Hydrologic Routing Techniques

Hydrologic routing employs the use of the continuity equation and either an analytical or an empirical relationship between storage within the reach and discharge at the outlet. In its simplest form, the continuity equation can be written as inflow minus outflow equals the rate of change of storage within the reach:

$$I - O = \frac{\Delta S}{\Delta t} \tag{9-10}$$

where

I = the average inflow to the reach during Δt

O = the average outflow from the reach during Δt

S = storage within the reach

a. Modified puls reservoir routing.

(1) One of the simplest routing applications is the analysis of a floodwave that passes through an unregulated reservoir (Figure 9-2a). The inflow hydrograph is known, and it is desired to compute the outflow hydrograph from the reservoir. Assuming that all gate and spillway openings are fixed, a unique relationship between storage and outflow can be developed, as shown in Figure 9-2b.

(2) The equation defining storage routing, based on the principle of conservation of mass, can be written in approximate form for a routing interval Δt. Assuming the subscripts "1" and "2" denote the beginning and end of the routing interval, the equation is written as follows:

$$\frac{O_1 + O_2}{2} = \frac{I_1 + I_2}{2} - \frac{S_2 - S_1}{\Delta t} \tag{9-11}$$

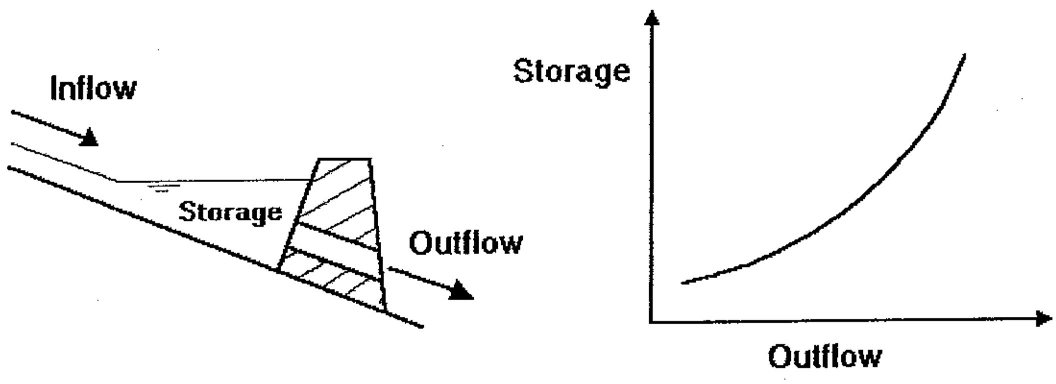


Figure 9-2. Reservoir storage routing

The known values in this equation are the inflow hydrograph and the storage and discharge at the beginning of the routing interval. The unknown values are the storage and discharge at the end of the routing interval. With two unknowns (O_2 and S_2) remaining, another relationship is required to obtain a solution. The storage-outflow relationship is normally used as the second equation. How that relationship is derived is what distinguishes various storage routing methods.

(3) For an uncontrolled reservoir, outflow and water in storage are both uniquely a function of lake elevation. The two functions can be combined to develop a storage-outflow relationship, as shown in Figure 9-3. Elevation-discharge relationships can be derived directly from hydraulic equations. Elevation-storage relationships are derived through the use of topographic maps. Elevation-area relationships are computed first, then either average end-area or conic methods are used to compute volumes.

(4) The storage-outflow relationship provides the outflow for any storage level. Starting with a nearly empty reservoir, the outflow capability would be minimal. If the inflow is less than the outflow capability, the water would flow through. During a flood, the inflow increases and eventually exceeds the outflow capability. The difference between inflow and outflow produces a change in storage. In Figure 9-4, the difference between the inflow and the outflow (on the rising side of the outflow hydrograph) represents the volume of water entering storage.

(5) As water enters storage, the outflow capability increases because the pool level increases. Therefore, the outflow increases. This increasing outflow with increasing water in storage continues until the reservoir reaches a maximum level. This will occur the moment that the outflow equals the inflow, as shown in Figure 9-4. Once the outflow becomes greater than the inflow, the storage level will begin dropping. The difference between the outflow and the inflow hydrograph on the recession side reflects water withdrawn from storage.

(6) The modified puls method applied to reservoirs consists of a repetitive solution of the continuity equation. It is assumed that the reservoir water surface remains horizontal, and therefore, outflow is a unique function of reservoir storage. The continuity equation, Equation 9-11, can be manipulated to get both of the unknown variables on the left-hand side of the equation:

$$\left(\frac{S_2}{\Delta t} + \frac{O_2}{2}\right) = \left(\frac{S_1}{\Delta t} + \frac{O_1}{2}\right) - O_1 + \frac{I_1 + I_2}{2} \qquad (9\text{-}12)$$

Since I is known for all time steps, and O_1 and S_1 are known for the first time step, the right-hand side of the equation can be calculated. The left-hand side of the equation can be solved by trial and error. This is accomplished by assuming a value for either S_2 or O_2, obtaining the corresponding value from the storage-outflow relationship, and then iterating until Equation 9-12 is satisfied.

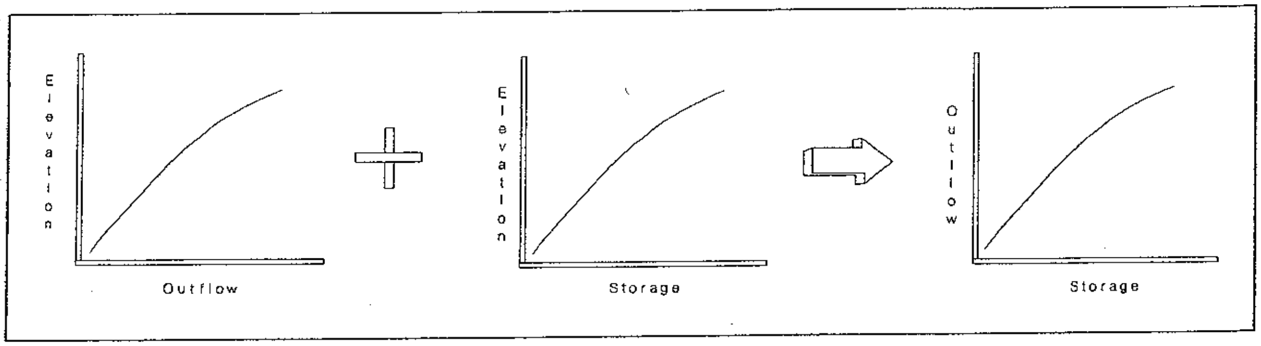


Figure 9-3. Reservoir storage-outflow curve

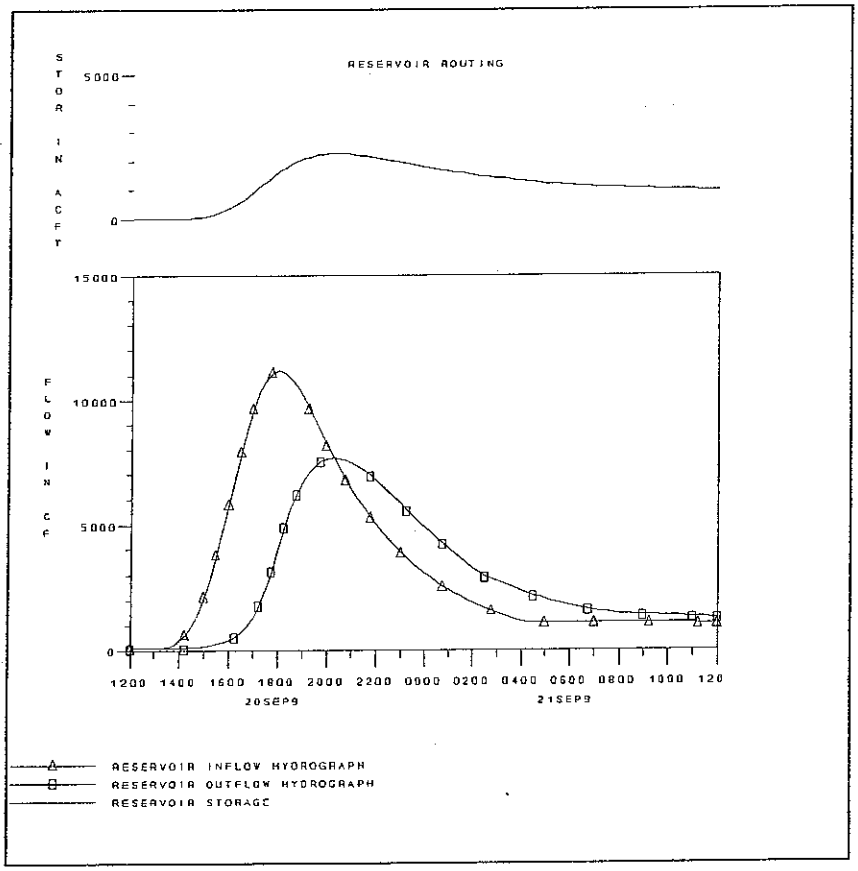


Figure 9-4. Reservoir routing example

Rather than resort to this iterative procedure, a value of Δt is selected and points on the storage-outflow curve are replotted as the "storage-indication" curve shown in Figure 9-5. This graph allows for a direct determination of the outflow (O_2) once a value of storage indication ($S_2/\Delta t + O_2/2$) has been calculated from Equation 9-12 (Viessman et al. 1977). The numerical integration of Equation 9-12 and Figure 9-5 is illustrated as an example in Table 9-1. The stepwise procedure for applying the modified puls method to reservoirs can be summarized as follows:

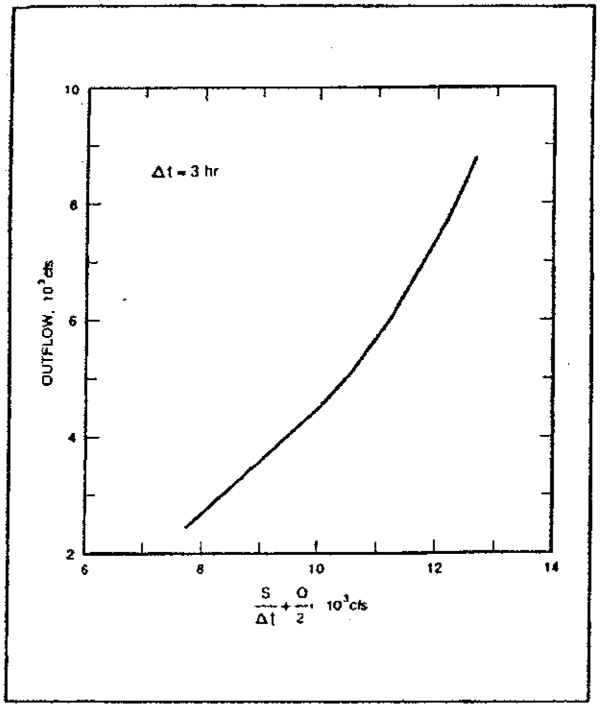


Figure 9-5. Storage-indication curve

(a) Determine a composite discharge rating curve for all of the reservoir outlet structures.

(b) Determine the reservoir storage that corresponds with each elevation on the rating curve for reservoir outflow.

(c) Select a time step and construct a storage-indication versus outflow curve $[(S/\Delta t) + (O/2)]$ versus O.

(d) Route the inflow hydrograph through the reservoir based on Equation 9-12 and the storage-indication curve.

(e) Compare the results with historical events to verify the model.

b. Modified puls channel routing. Routing in natural rivers is complicated by the fact that storage in a river reach is not a function of outflow alone. During the passing of a floodwave, the water surface in a channel is not uniform. The storage and water surface slope within a river reach, for a given outflow, is greater during the rising stages of a floodwave than during the falling (Figure 9-6). Therefore, the relationship between storage and discharge at the outlet of a channel is not a unique relationship, rather it is a looped relationship. An example storage-discharge function for a river is shown in Figure 9-7.

(1) Application of the modified puls method to rivers. To apply the modified puls method to a channel routing problem, the storage within the river reach is approximated with a series of "cascading reservoirs" (Figure 9-8). Each reservoir is assumed to have a level pool and, therefore, a unique storage-discharge relationship. The cascading reservoir approach is capable of approximating the looped storage-outflow effect when evaluating the river reach as a whole. The rising and falling floodwave is simulated with different storage levels in the cascade of reservoirs, thus producing a looped storage-outflow function for the total river reach. This is depicted graphically in Figure 9-9.

(2) Determination of the storage-outflow relationship.

(a) Determining the storage-outflow relationship for a river reach is a critical part of the modified puls procedure. In river reaches, storage-outflow relationships can be determined from one of the following:

- steady-flow profile computations,
- observed water surface profiles,
- normal-depth calculations,
- observed inflow and outflow hydrographs, and
- optimization techniques applied to observed inflow and outflow hydrographs.

(b) Steady-flow water surface profiles, computed over a range of discharges, can be used to determine storage-outflow relationships in a river reach (Figure 9-10). In this illustration, a known hydrograph at A is to be routed to location B. The storage-outflow relationship required for routing is determined by computing a series of water surface profiles, corresponding to a range of discharges. The range of discharges should encompass the range of flows that will be routed through the river reach. The storage volumes are computed by multiplying the cross-sectional area, under a specific flow profile, by the channel reach lengths. Volumes are calculated for each flow profile and then plotted against