Examples: Taken from Open-channel hydraulics, by Ven Te Chow, McGraw-Hill.

$$S_f = C_f \frac{U^2}{g y} = \frac{m^2 U^2}{\text{[redacted]} R^{5/3}}$$

Example of explicit method

(6)

Table 10-4. Computation of the Flow Profile by the Direct Step Method for Example 10-7

$Q = 400$ cfs $n = 0.025$ $S_0 = 0.0016$ $\alpha = 1.10$ $y_c = 2.22$ ft $y_n = 3.36$ ft

y (1)	A (2)	R (3)	$R^{4/3}$ (4)	V (5)	$\alpha V^2/2g$ (6)	E (7)	ΔE (8)	S_f (9)	$\bar{S}_f$ (10)	$S_0 - \bar{S}_f$ (11)	Δx (12)	x (13)
5.00	150.00	3.54	5.40	2.667	0.1217	5.1217	......	0.000370				
4.80	142.08	3.43	5.17	2.819	0.1356	4.9356	0.1861	0.000433	0.000402	0.001198	155	155
4.60	134.32	3.31	4.94	2.979	0.1517	4.7517	0.1839	0.000507	0.000470	0.001130	163	318
4.40	126.72	3.19	4.70	3.156	0.1706	4.5706	0.1811	0.000598	0.000553	0.001047	173	491
4.20	119.28	3.08	4.50	3.354	0.1925	4.3925	0.1781	0.000705	0.000652	0.000948	188	679
4.00	112.00	2.96	4.25	3.572	0.2184	4.2184	0.1741	0.000850	0.000778	0.000822	212	891
3.80	104.88	2.84	4.02	3.814	0.2490	4.0490	0.1694	0.001020	0.000935	0.000665	255	1,146
3.70	101.38	2.77	3.88	3.948	0.2664	3.9664	0.0826	0.001132	0.001076	0.000524	158	1,304
3.60	97.92	2.71	3.78	4.085	0.2856	3.8856	0.0808	0.001244	0.001188	0.000412	196	1,500
3.55	96.21	2.68	3.72	4.158	0.2958	3.8458	0.0398	0.001310	0.001277	0.000323	123	1,623
3.50	94.50	2.65	3.66	4.233	0.3067	3.8067	0.0391	0.001382	0.001346	0.000254	154	1,777
3.47	93.48	2.63	3.63	4.278	0.3131	3.7831	0.0236	0.001427	0.001405	0.000195	121	1,898
3.44	92.45	2.61	3.59	4.326	0.3202	3.7602	0.0229	0.001471	0.001449	0.000151	152	2,050
3.42	91.80	2.60	3.57	4.357	0.3246	3.7446	0.0156	0.001500	0.001486	0.000114	137	2,187
3.40	91.12	2.59	3.55	4.388	0.3292	3.7292	0.0154	0.001535	0.001518	0.000082	188	2,375

Another example of explicit method

(7)

TABLE 10-5. COMPUTATION OF THE FLOW PROFILE FOR EXAMPLE 10-8 BY THE DIRECT STEP METHOD

$Q = 252$ cfs $n = 0.012$ $S_0 = 0.02$ $\alpha = 1.0$ $y_c = 4.35$ ft $y_n = 2.60$ ft

y/D	y	A	R	$R^{2/3}$	V	$\alpha V^2/2g$	E	ΔE	S_f	$\bar{S}_f$	$S_0 - \bar{S}_f$	Δx	x
0.725	4.35	21.95	1.794	2.180	11.48	2.048	6.398		0.00392				
0.70	4.20	21.13	1.777	2.154	11.93	2.211	6.411	0.013	0.00429	0.00411	0.01589	0.8	0.8
0.65	3.90	19.45	1.728	2.073	12.96	2.609	6.509	0.098	0.00525	0.00477	0.01523	6.4	7.2
0.60	3.60	17.71	1.666	1.976	14.23	3.145	6.745	0.236	0.00666	0.00596	0.01404	16.8	24.0
0.55	3.30	15.93	1.590	1.855	15.85	3.901	7.201	0.456	0.00880	0.00773	0.01227	37.2	61.2
0.50	3.00	14.13	1.500	1.717	17.85	4.947	7.947	0.746	0.01202	0.01041	0.00959	77.8	139.0
0.48	2.88	13.42	1.460	1.656	18.76	5.465	8.345	0.398	0.01378	0.01290	0.00710	56.1	195.1
0.47	2.82	13.06	1.440	1.626	19.30	5.785	8.605	0.260	0.01486	0.01432	0.00568	45.8	240.9
0.46	2.76	12.70	1.420	1.596	19.85	6.119	8.879	0.274	0.01600	0.01543	0.00457	60.0	300.9

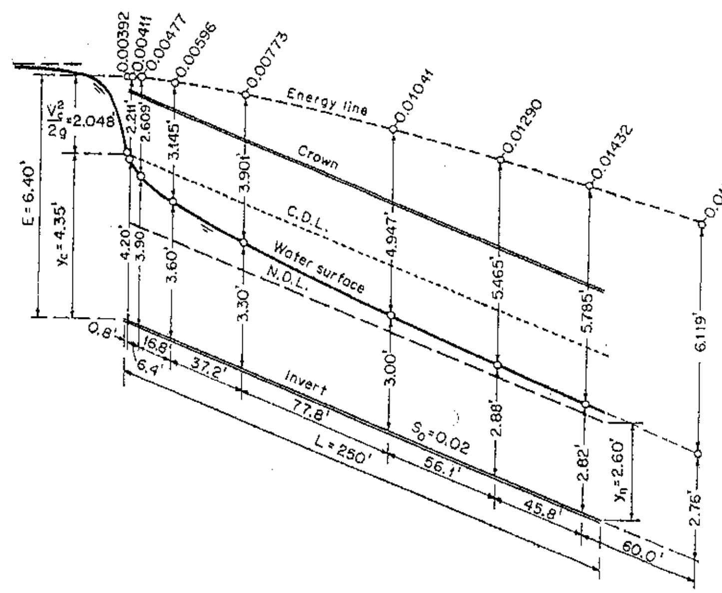

FIG. 10-7. An $S2$ flow profile computed by the direct step method.

Implicit method

(8)

Solution. The step computations are arranged in tabular form, as shown in Table 10-6. Values in each column of the table are explained as follows:

Col. 1. Section identified by station number such as "station 1 + 55." The location of the stations is fixed at the distances determined in Example 10-7 in order to compare the procedure with that of the direct step method.

Col. 2. Water-surface elevation at the station. A trial value is first entered in this column; this will be verified or rejected on the basis of the computations made in the remaining columns of the table. For the first step, this elevation must be given or assumed. Since the elevation of the dam site is 600 m.s.l. and the height of the dam is 5 ft, the first entry is 605.00 m.s.l. When the trial value in the second step has been verified, it becomes the basis for the verification of the trial value in the next step, and so on.

Col. 3. Depth of flow in ft, corresponding to the water-surface elevation in col. 2. For instance, the depth of flow at station 1 + 55 is equal to water-surface elevation minus elevation at the dam site minus (distance from the dam site times bed slope), or $605.048 - 600.000 - 155 \times 0.0016 = 4.80$ ft.

Col. 4. Water area corresponding to y in col. 3

Col. 5. Mean velocity equal to the given discharge 400 cfs divided by the water area in col. 4

Col. 6. Velocity head in ft, corresponding to the velocity in col. 5

Col. 7. Total head computed by Eq. (10-47), equal to the sum of Z in col. 2 and the velocity head in col. 6

Col. 8. Hydraulic radius in ft, corresponding to y in col. 3

Col. 9. Four-thirds power of the hydraulic radius

Col. 10. Friction slope computed by Eq. (9-8), with $n = 0.025$, V from col. 5, and $R^{4/3}$ from col. 9

Col. 11. Average friction slope through the reach between the sections in each step, approximately equal to the arithmetic mean of the friction slope just computed in col. 10 and that of the previous step

Col. 12. Length of the reach between the sections, equal to the difference in station numbers between the stations

Col. 13. Friction loss in the reach, equal to the product of the values in cols. 11 and 12.

Col. 14. Eddy loss in the reach, equal to zero

Col. 15. Elevation of the total head in ft. This is computed by Eq. (10-49), that is, by adding the values of h_f and h_e in cols. 13 and 14 to the elevation at the lower end of the reach, which is found in col. 15 of the previous reach. If the value so obtained does not agree closely with that entered in col. 7, a new trial value of the water-surface elevation is assumed, and so on, until agreement is obtained. The value that leads to agreement is the correct water-surface elevation. The computation may then proceed to the next step. The computed flow profile is practically identical with that obtained by the graphical-integration method shown in Fig. 10-3.

10-5. Computation of a Family of Flow Profiles. In previous articles methods were described for determining a single flow profile. Frequently, several flow profiles, or a family of flow profiles, are desired for various conditions of stage and discharge. An example of this type of problem is the determination of the economical height of a dam, where the initial elevation is indeterminate and, hence, a number of flow profiles may have to be computed for the same discharge with different assumed

Implicit

TABLE 10-6. COMPUTATION OF THE FLOW PROFILE FOR EXAMPLE 10-9 BY THE STANDARD STEP METHOD

$Q = 400$ cfs $\quad n = 0.025 \quad S_0 = 0.0016 \quad \alpha = 1.10 \quad h_e = 0 \quad y_c = 2.22$ ft $\quad y_n = 3.36$ ft

Station (1)	Z (2)	y (3)	A (4)	V (5)	$\alpha V^2/2g$ (6)	H (7)	R (8)	$R^{4/3}$ (9)	S_f (10)	$\bar{S}_f$ (11)	Δx (12)	h_f (13)	h_e (14)	H (15)
0 + 00	605.000	5.00	150.00	2.667	0.1217	605.122	3.54	5.40	0.000370	....	...	.....	...	605.122
1 + 55	605.048	4.80	142.08	2.819	0.1356	605.184	3.43	5.17	0.000433	0.000402	155	0.062	0	605.184
3 + 18	605.109	4.60	134.32	2.979	0.1517	605.261	3.31	4.92	0.000507	0.000470	163	0.077	0	605.261
4 + 91	605.186	4.40	126.72	3.156	0.1706	605.357	3.19	4.70	0.000598	0.000553	173	0.096	0	605.357
6 + 79	605.286	4.20	119.28	3.354	0.1925	605.479	3.08	4.50	0.000705	0.000652	188	0.122	0	605.479
8 + 91	605.426	4.00	112.00	3.572	0.2184	605.644	2.96	4.25	0.000850	0.000778	212	0.165	0	605.644
11 + 46	605.633	3.80	104.88	3.814	0.2490	605.882	2.84	4.02	0.001020	0.000935	255	0.238	0	605.882
13 + 04	605.786	3.70	101.38	3.948	0.2664	606.052	2.77	3.88	0.001132	0.001076	158	0.170	0	606.052
15 + 00	605.999	3.60	97.92	4.085	0.2856	606.285	2.71	3.78	0.001244	0.001188	196	0.233	0	606.285
16 + 23	606.146	3.55	96.21	4.158	0.2958	606.442	2.68	3.72	0.001310	0.001277	123	0.157	0	606.442
17 + 77	606.343	3.50	94.50	4.233	0.3067	606.650	2.65	3.66	0.001382	0.001346	154	0.208	0	606.650
18 + 98	606.507	3.47	93.48	4.278	0.3131	606.820	2.63	3.63	0.001427	0.001405	121	0.170	0	606.820
20 + 50	606.720	3.44	92.45	4.326	0.3202	607.040	2.61	3.59	0.001471	0.001449	152	0.220	0	607.040
21 + 87	606.919	3.42	91.80	4.357	0.3246	607.244	2.60	3.57	0.001500	0.001486	137	0.204	0	607.244
23 + 75	607.201	3.40	91.12	4.388	0.3292	607.530	2.59	3.55	0.001535	0.001518	188	0.286	0	607.530

9

HAND OUT 17: Hydrologic routing (Chapter 6 of our syllabus). Source: Mays, L. (2006). *"Water resources engineering."* John Wiley and Sons.

Chapter 9

Reservoir and Stream Flow Routing

9.1 ROUTING

Figure 9.1.1 illustrates how stream flow increases as the *variable source area* extends into the drainage basin. The variable source area is the area of the watershed that is actually contributing flow to the stream at any point. The variable source area expands during rainfall and contracts thereafter.

Flow routing is the procedure to determine the time and magnitude of flow (i.e., the flow hydrograph) at a point on a watercourse from known or assumed hydrographs at one or more points upstream. If the flow is a flood, the procedure is specifically known as flood routing. Routing by lumped system methods is called *hydrologic (lumped) routing*, and routing by distributed systems methods is called *hydraulic (distributed) routing*.

For hydrologic routing, input $I(t)$, output $Q(t)$, and storage $S(t)$ as functions of time are related by the continuity equation (3.2.10)

$$\frac{dS}{dt} = I(t) - Q(t) \tag{9.1.1}$$

Even if an inflow hydrograph $I(t)$ is known, equation (9.1.1) cannot be solved directly to obtain the outflow hydrograph $Q(t)$, because both Q and S are unknown. A second relationship, or storage function, is required to relate S, I, and Q; coupling the storage function with the continuity equations provides a solvable combination of two equations and two unknowns.

The specific form of the storage function depends on the nature of the system being analyzed. In reservoir routing by the level pool method (Section 9.2), storage is a nonlinear function of Q, $S = f(Q)$ and the function $f(Q)$ is determined by relating reservoir storage and outflow to reservoir water level. In the Muskingum method (Section 9.3) for flow routing in channels, storage is linearly related to I and Q.

The effect of storage is to redistribute the hydrograph by shifting the centroid of the inflow hydrograph to the position of that of the outflow hydrograph in a *time of redistribution*. In very long channels, the entire flood wave also travels a considerable distance and the centroid of its hydrograph may then be shifted by a time period longer than the time of redistribution. This additional time may be considered the *time of translation*. The total time of flood movement between the centroids of the inflow and outflow hydrographs is equal to the sum of the time of redistribution and the time of translation. The process of redistribution modifies the shape of the hydrograph, while translation changes its position.

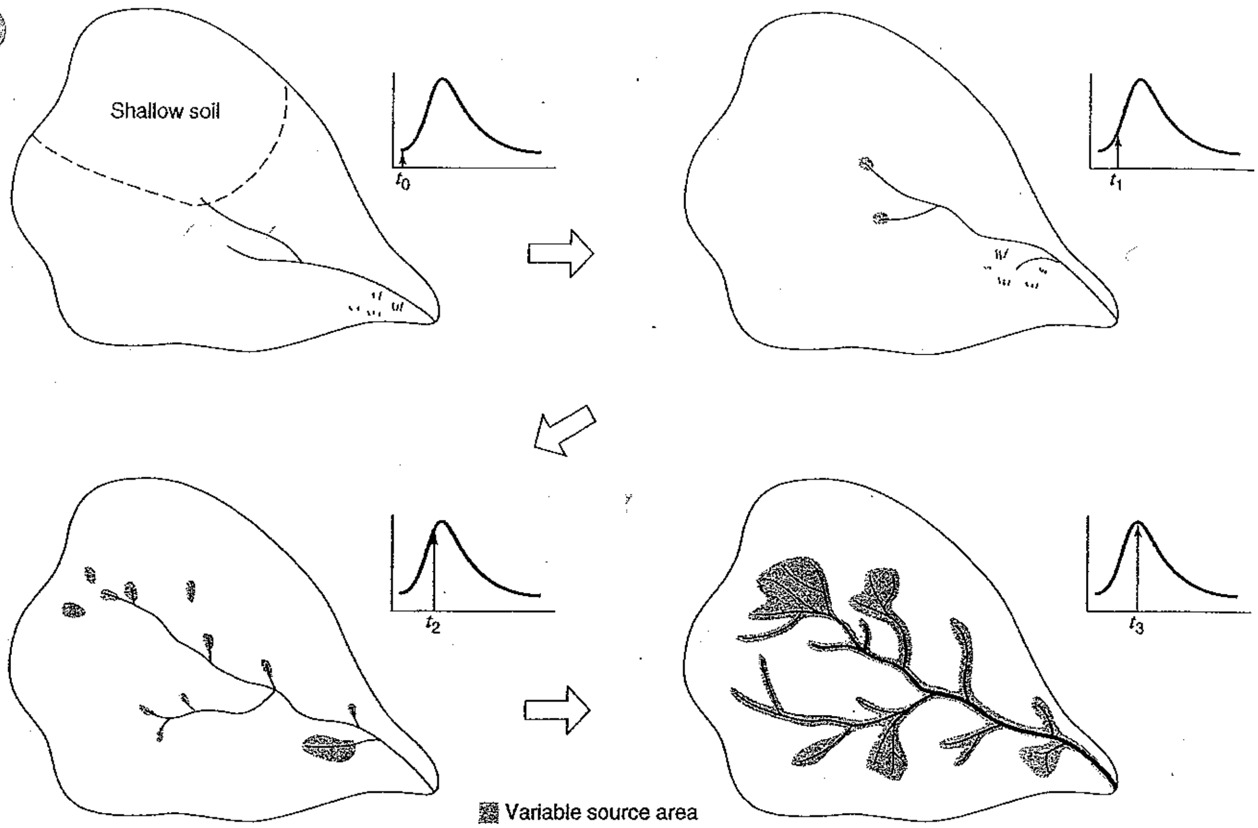

Figure 9.1.1 The small arrows in the hydrographs show how streamflow increases as the variable source extends into swamps, shallow soils, and ephemeral channels. The process reverses as streamflow declines (from Hewlett (1982)).

9.2 HYDROLOGIC RESERVOIR ROUTING

Level pool routing is a procedure for calculating the outflow hydrograph from a reservoir assuming a horizontal water surface, given its inflow hydrograph and storage-outflow characteristics. Equation (9.1.1) can be expressed in the in finite-difference form to express the change in storage over a time interval (see Figure 9.2.1) as

$$S_{j+1} - S_j = \frac{I_j + I_{j+1}}{2}\Delta t - \frac{Q_j + Q_{j+1}}{2}\Delta t \tag{9.2.1}$$

The inflow values at the beginning and end of the jth time interval are I_j and I_{j+1}, respectively, and the corresponding values of the outflow are Q_j and Q_{j+1}. The values of I_j and I_{j+1} are prespecified. The values of Q_j and S_j are known at the jth time interval from calculations for the previous time interval. Hence, equation (9.2.1) contains two unknowns, Q_{j+1} and S_{j+1}, which are isolated by multiplying (9.1.1) through by $2/\Delta t$, and rearranging the result to produce:

$$\left[\frac{2S_{j+1}}{\Delta t} + Q_{j+1}\right] = (I_j + I_{j+1}) + \left[\frac{2S_j}{\Delta t} - Q_j\right] \tag{9.2.2}$$

In order to calculate the outflow Q_{j+1}, a storage-outflow function relating $2S/\Delta t + Q$ and Q is needed. The method for developing this function using elevation-storage and elevation-outflow relationships is shown in Figure 9.2.2. The relationship between water surface elevation and reservoir storage can be derived by planimetering topographic maps or from field surveys. The elevation-discharge relation is derived from hydraulic equations relating head and discharge for

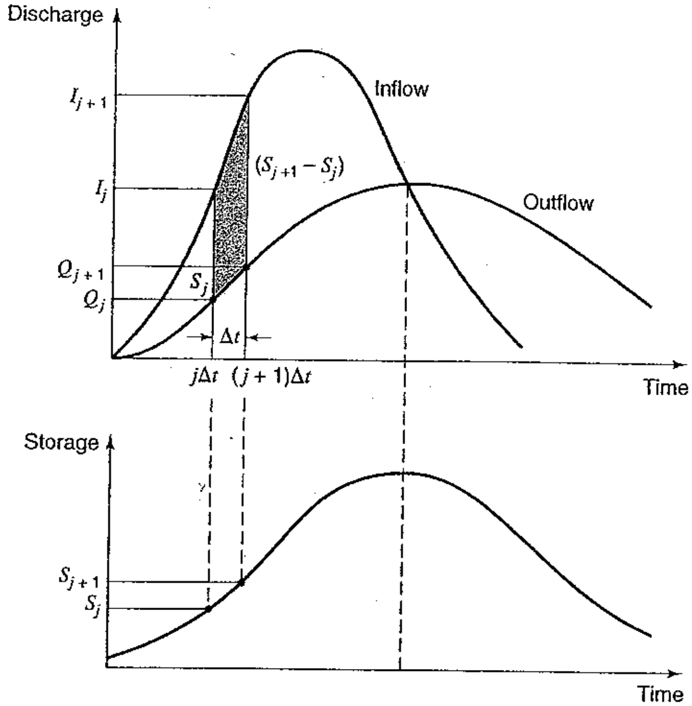

Figure 9.2.1 Change of storage during a routing period Δt.

various types of spillways and outlet works. (See Chapter 17.) The value of Δt is taken as the time interval of the inflow hydrograph. For a given value of water surface elevation, the values of storage S and discharge Q are determined (parts (*a*) and (*b*) of Figure 9.2.2), and then the value of $2S/\Delta t + Q$ is calculated and plotted on the horizontal axis of a graph with the value of the outflow Q on the vertical axis (part (*c*) of Figure 9.2.2).

In routing the flow through time interval j, all terms on the right side of equation (9.2.2) are known, and so the value of $2S_{j+1}/\Delta t + Q_{j+1}$ can be computed. The corresponding value of Q_{j+1} can be determined from the storage-outflow function $2S/\Delta t + Q$ versus Q, either graphically or by linear interpolation of tabular values. To set up the data required for the next time interval, the value of $(2S_{j+1}/\Delta t - Q_{j+1})$ is calculated using

$$\left[\frac{2S_{j+1}}{\Delta t} - Q_{j+1}\right] = \left[\frac{2S_{j+1}}{\Delta t} + Q_{j+1}\right] - 2Q_{j+1} \tag{9.2.3}$$

The computation is then repeated for subsequent routing periods.

EXAMPLE 9.2.1

Consider a 2-acre stormwater detention basin with vertical walls. The triangular inflow hydrograph increases linearly from zero to a peak of 60 cfs at 60 min and then decreases linearly to a zero discharge at 180 min. Route the inflow hydrograph through the detention basin using the head-discharge relationship for the 5-ft diameter pipe spillway in columns (1) and (2) of Table 9.2.1. The pipe is located at the bottom of the basin. Assuming the basin is initially empty, use the level pool routing procedure with a 10-min time interval to determine the maximum depth in the detention basin.

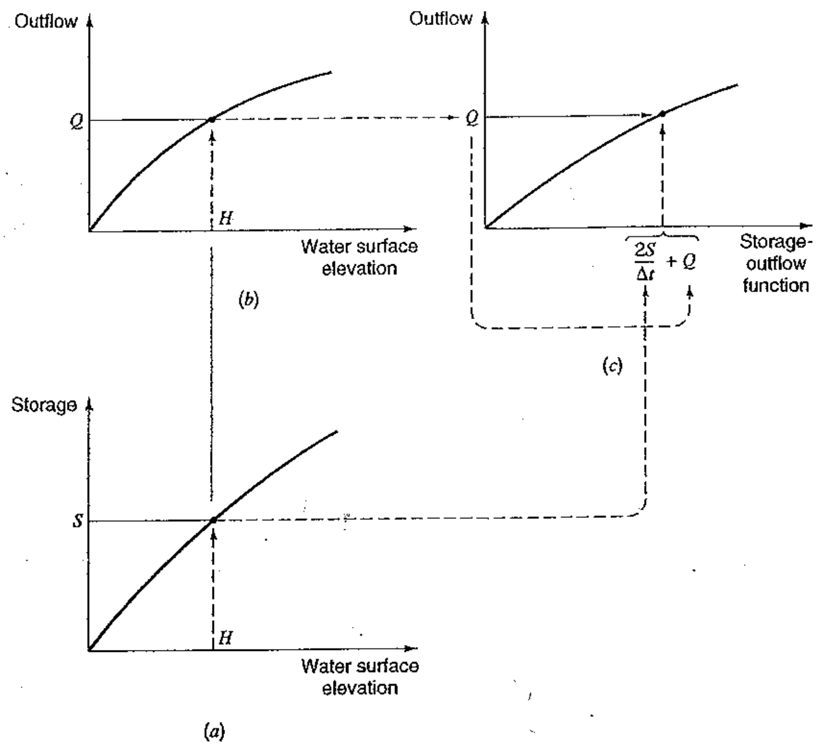

Figure 9.2.2 Development of the storage-outflow function for level pool routing on the basis of storage-elevation-outflow curves (from Chow et al. (1988)).

Table 9.2.1 Elevation-Discharge-Storage Data for Example 9.2.1

1	2	3	4
Head H (ft)	Discharge Q (cfs)	Storage S (ft^3)	$\frac{2S}{\Delta t}+Q$ (cfs)
0.0	0	0	.00
0.5	3	43,500	148.20
1.0	8	87,120	298.40
1.5	17	130,680	452.60
2.0	30	174,240	610.80
2.5	43	217,800	769.00
3.0	60	261,360	931.20
3.5	78	304,920	1094.40
4.0	97	348,480	1258.60
4.5	117	392,040	1423.80
5.0	137	435,600	1589.00

SOLUTION

The inflow hydrograph and the head-discharge (columns 1 and 3) and discharge-storage (columns 2 and 3) relationships are used to determine the routing relationship in Table 9.2.1. A routing interval of 10 min is used to determine the routing relationship $2S/\Delta t + Q$ vs. Q, which is columns 2 and 4 in Table 9.2.1. The routing computations are presented in Table 9.2.2, with the sequence of computations indicated by the arrows. These computations are carried out using equation (9.2.3). For the first time interval, $S_1 = Q_1 = 0$ because the reservoir is empty at $t = 0$; then $(2S_1/\Delta t - Q_1) = 0$. The value of the storage-outflow function at the end of the time interval is

$$\left[\frac{2S_2}{\Delta t}+Q_2\right]=(I_1+I_2)+\left[\frac{2S_1}{\Delta t}-Q_1\right]=(0+10)+0=10$$

The value of Q_2 is determined using linear interpolation, so that

$$Q_2=0+\frac{(3-0)}{(148.2-0)}(10-0)=0.2\text{ cfs}$$

With $Q_2 = 0.2$, then $2S_2/\Delta t - Q_2$ for the next iteration is

$$\left[\frac{2S_2}{\Delta t}-Q_2\right]=\left[\frac{2S_2}{\Delta t}+Q_2\right]-2Q_2=10-2(0.2)=9.6\text{ cfs}$$

The computation now proceeds to the next time interval. Refer to Table 9.2.1 for the remaining computations.

Table 9.2.2 Routing of Flow Through Detention Reservoir by the Level Pool Method (example 9.2.1)

Time t (min)	Inflow I_j (cfs)	$I_j + I_{j+1}$ (cfs)	$\frac{2S_j}{\Delta t} - Q_j$ (cfs)	$\frac{2S_{j+1}}{\Delta t} + Q_{j+1}$ (cfs)	Outflow (cfs)
.00	.00				.00
10.00	10.00	10.00	.00	10.00	.20
20.00	20.00	30.00	9.60	39.60	.80
30.00	30.00	50.00	37.99	87.99	1.78
40.00	40.00	70.00	84.43	154.43	3.21
50.00	50.00	90.00	148.01	238.01	5.99
60.00	60.00	110.00	226.04	336.04	10.20
70.00	55.00	115.00	315.64	430.64	15.72
80.00	50.00	105.00	399.21	504.21	21.24
90.00	45.00	95.00	461.72	556.72	25.56
100.00	40.00	85.00	505.61	590.61	28.34
110.00	35.00	75.00	533.93	608.93	29.85
120.00	30.00	65.00	549.24	614.24	30.28
130.00	25.00	55.00	553.67	608.67	29.83
140.00	20.00	45.00	549.02	594.02	28.62
150.00	15.00	35.00	536.78	571.78	26.79
160.00	10.00	25.00	518.19	543.19	24.44
170.00	5.00	15.00	494.30	509.30	21.66
180.00	.00	5.00	465.98	470.98	18.51
190.00	.00	.00	433.96	433.96	15.91
200.00	.00	.00	402.14	402.14	14.05
210.00	.00	.00	374.03	374.03	12.41
220.00	.00	.00	349.20	349.20	10.97
230.00	.00	.00	327.27	327.27	9.69
240.00	.00	.00	307.90	307.90	8.55

9.3 HYDROLOGIC RIVER ROUTING

The *Muskingum method* is a commonly used hydrologic routing method that is based upon a variable discharge-storage relationship. This method models the storage volume of flooding in a river channel by a combination of wedge and prism storage (Figure 9.3.1). During the advance of a flood wave, inflow exceeds outflow, producing a wedge of storage. During the recession, outflow exceeds inflow, resulting in a negative wedge. In addition, there is a prism of storage that is formed by a volume of constant cross-section along the length of prismatic channel.

Assuming that the cross-sectional area of the flood flow is directly proportional to the discharge at the section, the *volume of prism storage* is equal to KQ, where K is a proportionality coefficient (approximate as the travel time through the reach), and the *volume of wedge storage* is equal to $KX(I - Q)$, where X is a weighting factor having the range $0 \le X \le 0.5$. The total storage is defined as the sum of two components,

$$S = KQ + KX(I - Q) \tag{9.3.1}$$

which can be rearranged to give the storage function for the Muskingum method

$$S = K[XI + (1 - X)Q)] \tag{9.3.2}$$

and represents a linear model for routing flow in streams.

The value of X depends on the shape of the modeled wedge storage. The value of X ranges from 0 for reservoir-type storage to 0.5 for a full wedge. When $X = 0$, there is no wedge and hence no backwater; this is the case for a level-pool reservoir. In natural streams, X is between 0 and 0.3, with a mean value near 0.2. Great accuracy in determining X may not be necessary because the results of the method are relatively insensitive to the value of this parameter. The parameter K is the time of travel of the flood wave through the channel reach. For hydrologic routing, the values of K and X are assumed to be specified and constant throughout the range of flow.

The values of storage at time j and $j + 1$ can be written, respectively, as

$$S_j = K[XI_j + (1 - X)Q_j] \tag{9.3.3}$$

$$S_{j+1} = K[XI_{j+1} + (1 - X)Q_{j+1}] \tag{9.3.4}$$

Using equations (9.3.3) and (9.3.4), the change in storage over time interval Δt is

$$S_{j+1} - S_j = K\{[XI_{j+1} + (1 - X)Q_{j+1}] - [XI_j + (1 - X)Q_j]\} \tag{9.3.5}$$

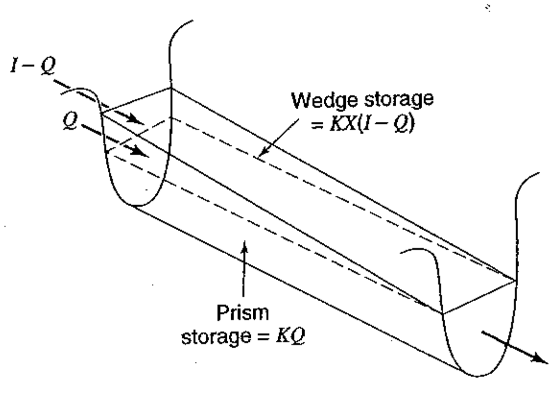

Figure 9.3.1 Prism and wedge storages in a channel reach.

The change in storage can also be expressed using equation (9.2.1). Combining equations (9.3.5) and (9.2.1) and simplifying gives

$$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j \tag{9.3.6}$$

which is the routing equation for the Muskingum method, where

$$C_1 = \frac{\Delta t - 2KX}{2K(1-X)+\Delta t} \tag{9.3.7}$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1-X)+\Delta t} \tag{9.3.8}$$

$$C_3 = \frac{2K(1-X) - \Delta t}{2K(1-X)+\Delta t} \tag{9.3.9}$$

Note that $C_1 + C_2 + C_3 = 1$.

The routing procedure can be repeated for several sub-reaches (N_{steps}) so that the total travel time through the reach is K. To insure that the method is computationally stable and accurate, the U.S. Army Corps of Engineers (1990) uses the following criterion to determine the number of routing reaches:

$$\frac{1}{2(1-X)} \leq \frac{K}{N_{\text{steps}}\Delta t} \leq \frac{1}{2X} \tag{9.3.10}$$

If observed inflow and outflow hydrographs are available for a river reach, the values of K and X can be determined. Assuming various values of X and using known values of the inflow and outflow, successive values of the numerator and denominator of the following expression for K, derived from equations (9.3.5) and (9.3.8), can be computed using

$$K = \frac{0.5\Delta t\left[(I_{j+1} + I_j) - (Q_{j+1} + Q_j)\right]}{X(I_{j+1} - I_j) + (1-X)(Q_{j+1} - Q_j)} \tag{9.3.11}$$

The computed values of the numerator (storage) and denominator (weighted discharges) are plotted for each time interval, with the numerator on the vertical axis and the denominator on the horizontal axis. This usually produces a graph in the form of a loop, as shown in Figure 9.3.2. The value of X that produces a loop closest to a single line is taken to be the correct value for the reach, and K, according to equation (9.3.11), is equal to the slope of the line. Since K is the time required for the incremental flood wave to traverse the reach, its value may also be estimated as the observed time of travel of peak flow through the reach.

EXAMPLE 9.3.1

The objective of this example is to determine K and X for the Muskingum routing method using the February 26 to March 4, 1929 data on the Tuscasawas River from Dover to Newcomerstown. This example is taken from the U.S. Army Corps of Engineers (1960) as used in Cudworth (1989). Columns 2 and 3 in Table 9.3.1 are the inflow and outflow hydrographs for the reach. The numerator and denominator of equation (9.3.11) were computed (for each time period) using four values of X = 0, 0.1, 0.2, and 0.3. The accumulated numerators are in column 9 and the accumulated denominators (weighted discharges) are in columns 11, 13, 15, and 17. In Figure 9.3.2, the accumulated numerator (storages) from column (9) are plotted against the corresponding accumulated denominator (weighted discharges) for each of the four X values. According to Figure 9.3.2, the best fit (linear relationship) appears to be for $X = 0.2$, which has a resulting $K = 1.0$. To perform a routing, K should equal Δt, so that if $\Delta t = 0.5$ day, as in this case, the reach should be subdivided into two equal reaches ($N_{\text{steps}} = 2$) and the value of K should be 0.5 day for each reach.

Table 9.3.1 Determination of Coefficients K and X for the Muskingum Routing Method. Tuscarawas River, Muskingum Basin, Ohio Reach from Dover to Newcomerstown, February 26 to March 4, 1929

(1) Date Δt = 0.5 day	(2) Inflow[1], ft³/s	(3) Outflow[2], ft³/s	(4) $I_2 + I_1$, ft³/s	(5) $O_2 + O_1$, ft³/s	(6) $I_2 - I_1$, ft³/s	(7) $O_2 - O_1$, ft³/s	(8) [3]N	(9) ΣN	X = 0: [4]D (10)	X = 0: ΣD (11)	X = 0.1: D (12)	X = 0.1: ΣD (13)	X = 0.2: D (14)	X = 0.2: ΣD (15)	X = 0.3: D (16)	X = 0.3: ΣD (17)
2-26-29 a.m.	2,200	2,000	16,700	9,000	12,300	5,000	1,900		5,000		5,700		6,500		7,200	
p.m.	14,500	7,000	42,900	18,700	13,900	4,700	6,100	1,900	4,700	5,000	5,600	5,700	6,500	6,500	7,500	7,200
2-27-29 a.m.	28,400	11,700	60,200	28,200	3,400	4,800	8,000	8,000	4,800	9,700	4,600	11,300	4,500	13,000	4,300	14,700
p.m.	31,800	16,500	61,500	40,500	−2,100	7,500	5,200	16,000	7,500	14,500	6,700	15,900	5,600	17,500	4,600	19,000
2-28-29 a.m.	29,700	24,00	55,000	53,100	−4,400	5,100	500	21,200	5,100	22,000	4,100	22,600	3,200	23,100	2,300	23,600
p.m.	25,300	29,100	45,700	57,500	−4,900	−700	−2,900	21,700	−700	27,100	−1,100	26,700	−1,500	26,300	−2,000	25,900
3-01-29 a.m.	20,400	28,400	36,700	52,200	−4,100	−4,600	−3,900	18,800	−4,600	26,400	−4,600	25,600	−4,500	24,800	−4,400	23,900
p.m.	16,300	23,800	28,900	43,200	−3,700	−4,400	−3,600	14,900	−4,400	21,800	−4,300	21,000	−4,300	20,300	−4,200	19,500
3-02-29 a.m.	12,600	19,400	21,900	34,700	−3,300	−4,100	−3,200	11,300	−4,100	17,400	−4,000	16.700	−3,900	16,000	−3,900	15,300
p.m.	9,300	15,300	16,000	26,500	−2,600	−4,100	−2,500	8,100	−4,100	13,300	−4,000	12,700	−3,800	12,100	−3,600	11,400
3-03-29 a.m.	6,700	11,200	11,700	19,400	−1,700	−3,000	−1,900	5,500	−3,000	9,200	−2,800	8,700	−2,800	8,300	−2,600	7,800
p.m.	5,000	8,200	9,100	14,600	−900	−1,800	−1,400	3,600	−1,800	6,200	−1,700	5,900	−1,600	5,500	−1,600	5,200
3-04-29 a.m.	4,100	6,400	7,700	11,600	−500	−1,200	−1,000	2,200	−1,200	4,400	−1,200	4,200	−1,100	3,900	−900	3,600
p.m.	3,600	5,200	6,000	9,800	−1,200	−600	−1,000	1,200	−600	3,200	−600	3,000	−700	2,800	−800	2,700
3-05-29 a.m.	2,400	4,600	—	—	—	—	—	200	—	2,600	—	2,400	—	2,100	—	1,900

[1]Inflow to reach was adjusted to equal volume of outflow.
[2]Outflow is the hydrograph at Newcomerstown.
[3]Numerator, N, is $\Delta t/2$, column (4) – column (5).
[4]Denominator, D, is column (7) + X[column (6) – column (7)].
Note: From plottings of column (9) versus columns (11), (13), (15), and (17), the plot giving the best fit is considered to define K and X.

$$K = \frac{\text{Numerator, } N}{\text{Denominator, } D} = \frac{0.5\Delta t\left[(I_2 + I_1) - (O_2 + O_1)\right]}{X(I_2 - I_1) + (1 - X)(O_2 - O_1)}$$

Source: Cudworth (1989).

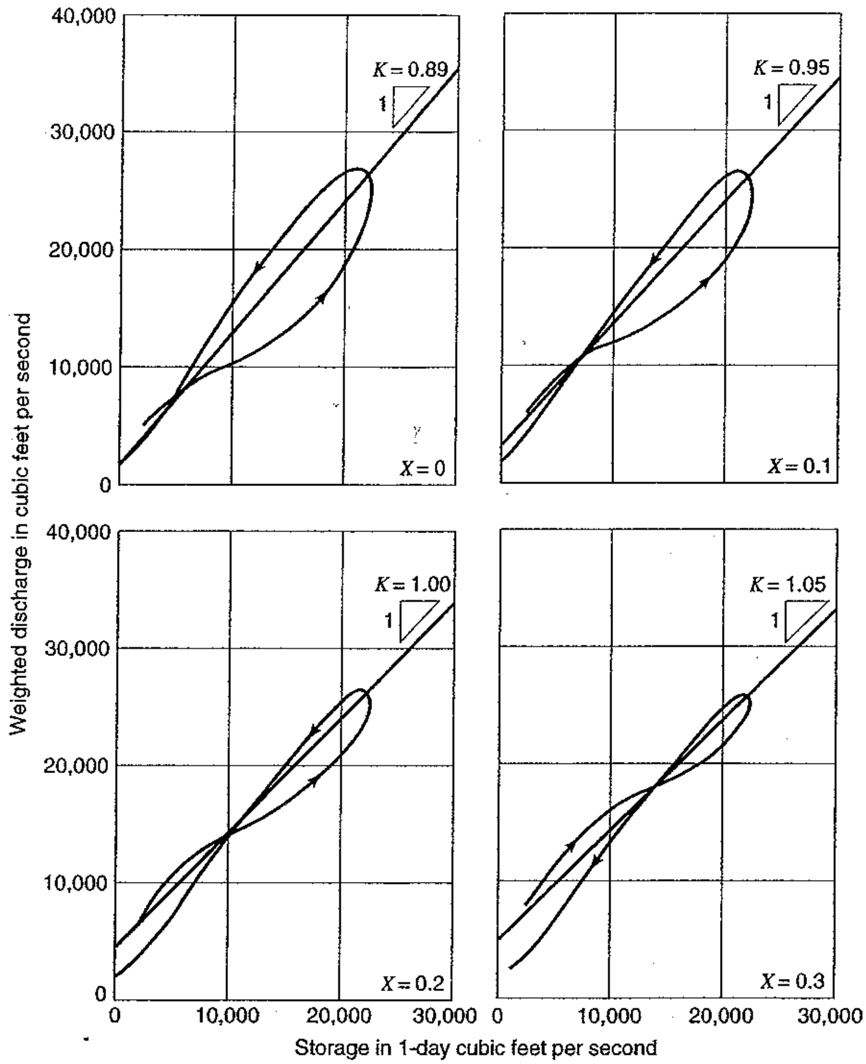

Figure 9.3.2 Typical valley storage curves.

EXAMPLE 9.3.2

Route the inflow hydrograph below using the Muskingum method; $\Delta t = 1$ hr, $X = 0.2$, $K = 0.7$ hrs.

Time (hrs)	0	1	2	3	4	5	6	7
Inflow (cfs)	0	800	2000	4200	5200	4400	3200	2500

Time (hrs)	8	9	10	11	12	13
Inflow (cfs)	2000	1500	1000	700	400	0

$$C_1 = \frac{1.0 - 2(0.7)(0.2)}{2(0.7)(1-0.2)+1.0} = 0.3396$$

$$C_2 = \frac{1.0 + 2(0.7)(0.2)}{2(0.7)(1-0.2)+1.0} = 0.6038$$

$$C_3 = \frac{2(0.7)(1-0.2) - 1.0}{2(0.7)(1-0.2)+1.0} = 0.0566$$

(Adapted from Masch (1984).)