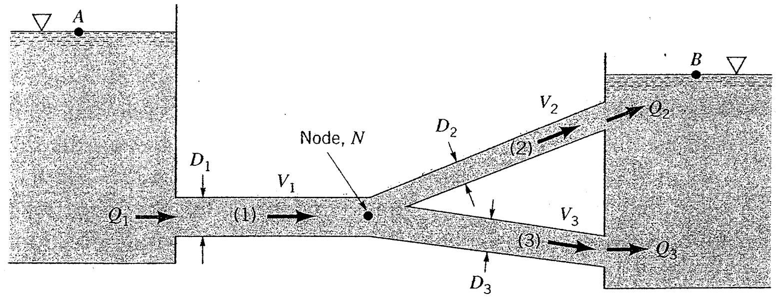


FIGURE 8.36 Multiple pipe loop system.

and

$$h_{L_1} = h_{L_2} = h_{L_3}$$

Again, the method of solution of these equations depends on what information is given and what is to be calculated.

Another type of multiple pipe system called a *loop* is shown in Fig. 8.36. In this case the flowrate through pipe (1) equals the sum of the flowrates through pipes (2) and (3), or $Q_1 = Q_2 + Q_3$. As can be seen by writing the energy equation between the surfaces of each reservoir, the head loss for pipe (2) must equal that for pipe (3), even though the pipe sizes and flowrates may be different for each. That is,

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_2}$$

for a fluid particle traveling through pipes (1) and (2), while

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B + h_{L_1} + h_{L_3}$$

for fluid that travels through pipes (1) and (3). These can be combined to give $h_{L_2} = h_{L_3}$. This is a statement of the fact that fluid particles that travel through pipe (2) and particles that travel through pipe (3) all originate from common conditions at the junction (or node, N) of the pipes and all end up at the same final conditions.

The three-reservoir problem can be quite complex.

The flow in a relatively simple looking multiple pipe system may be more complex than it appears initially. The branching system termed the *three-reservoir problem* shown in Fig. 8.37 is such a system. Three reservoirs at known elevations are connected together with three pipes of known properties (lengths, diameters, and roughnesses). The problem is to determine the flowrates into or out of the reservoirs. If valve (1) were closed, the fluid would flow from reservoir B to C, and the flowrate could be easily

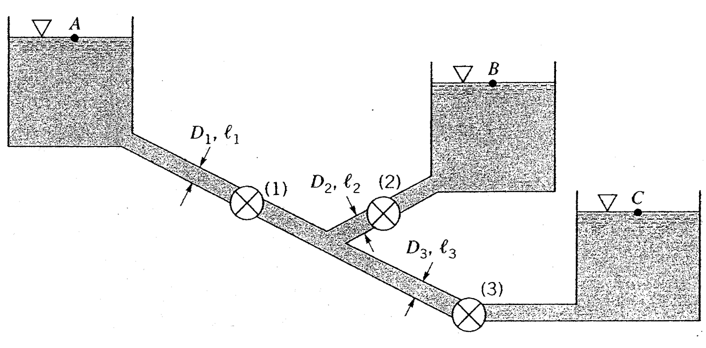


FIGURE 8.37 A three-reservoir system.

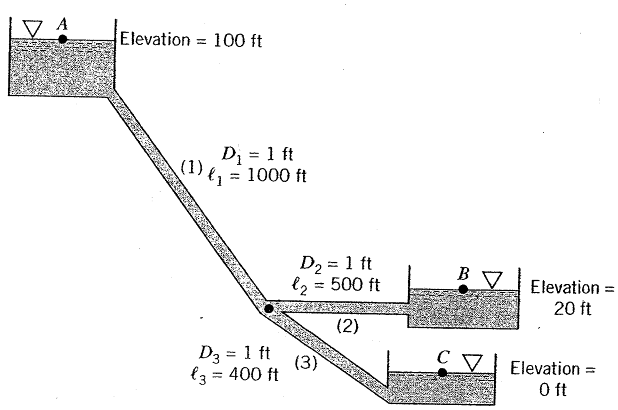
calculated. Similar calculations could be carried out if valves (2) or (3) were closed with the others open.

For some pipe systems, the direction of flow is not known a priori.

With all valves open, however, it is not necessarily obvious which direction the fluid flows. For the conditions indicated in Fig. 8.37, it is clear that fluid flows from reservoir A because the other two reservoir levels are lower. Whether the fluid flows into or out of reservoir B depends on the elevation of reservoirs B and C and the properties (length, diameter, roughness) of the three pipes. In general, the flow direction is not obvious, and the solution process must include the determination of this direction. This is illustrated in Example 8.14.

EXAMPLE 8.14 Three Reservoir, Multiple Pipe System

Three reservoirs are connected by three pipes as are shown in Fig. E8.14. For simplicity we assume that the diameter of each pipe is 1 ft, the friction factor for each is 0.02, and because of the large length-to-diameter ratio, minor losses are negligible. Determine the flowrate into or out of each reservoir.



■ FIGURE E8.14

SOLUTION

It is not obvious which direction the fluid flows in pipe (2). However, we assume that it flows out of reservoir B, write the governing equations for this case, and check our assumption. The continuity equation requires that $Q_1 + Q_2 = Q_3$, which, since the diameters are the same for each pipe, becomes simply

$$V_1 + V_2 = V_3 \tag{1}$$

The energy equation for the fluid that flows from A to C in pipes (1) and (3) can be written as

$$\frac{p_A}{\gamma} + \frac{V_A^2}{2g} + z_A = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3\frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

By using the fact that $p_A = p_C = V_A = V_C = z_C = 0$, this becomes

$$z_A = f_1\frac{\ell_1}{D_1}\frac{V_1^2}{2g} + f_3\frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

For the given conditions of this problem we obtain

$$100 \text{ ft} = \frac{0.02}{2(32.2 \text{ ft/s}^2)}\frac{1}{(1 \text{ ft})}[(1000 \text{ ft})V_1^2 + (400 \text{ ft})V_3^2]$$

or

$$322 = V_1^2 + 0.4V_3^2 \tag{2}$$

where V_1 and V_3 are in ft/s. Similarly the energy equation for fluid flowing from B and C is

$$\frac{p_B}{\gamma} + \frac{V_B^2}{2g} + z_B = \frac{p_C}{\gamma} + \frac{V_C^2}{2g} + z_C + f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g} + f_3\frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

or

$$z_B = f_2\frac{\ell_2}{D_2}\frac{V_2^2}{2g} + f_3\frac{\ell_3}{D_3}\frac{V_3^2}{2g}$$

For the given conditions this can be written as

$$64.4 = 0.5V_2^2 + 0.4V_3^2 \tag{3}$$

Equations 1, 2, and 3 (in terms of the three unknowns V_1, V_2, and V_3) are the governing equations for this flow, provided the fluid flows from reservoir B. It turns out, however, that there is no solution for these equations with positive, real values of the velocities. Although these equations do not appear to be complicated, there is no simple way to solve them directly. Thus, a trial-and-error solution is suggested. This can be accomplished as follows. Assume a value of $V_1 > 0$, calculate V_3 from Eq. 2, and then V_2 from Eq. 3. It is found that the resulting V_1, V_2, V_3 trio does not satisfy Eq. 1 for any value of V_1 assumed. There is no solution to Eqs. 1, 2, and 3 with real, positive values of V_1, V_2, and V_3. Thus, our original assumption of flow out of reservoir B must be incorrect.

To obtain the solution, assume the fluid flows into reservoirs B and C and out of A. For this case the continuity equation becomes

$$Q_1 = Q_2 + Q_3$$

or

$$V_1 = V_2 + V_3 \quad (4)$$

Application of the energy equation between points A and B and A and C gives

$$z_A = z_B + f_1 \frac{\ell_1}{D_1} \frac{V_1^2}{2g} + f_2 \frac{\ell_2}{D_2} \frac{V_2^2}{2g}$$

and

$$z_A = z_C + f_1 \frac{\ell_1}{D_1} \frac{V_1^2}{2g} + f_3 \frac{\ell_3}{D_3} \frac{V_3^2}{2g}$$

which, with the given data, become

$$258 = V_1^2 + 0.5\, V_2^2 \quad (5)$$

and

$$322 = V_1^2 + 0.4\, V_3^2 \quad (6)$$

Equations 4, 5, and 6 can be solved as follows. By subtracting Eq. 5 from 6 we obtain

$$V_3 = \sqrt{160 + 1.25 V_2^2}$$

Thus, Eq. 5 can be written as

$$258 = (V_2 + V_3)^2 + 0.5 V_2^2$$
$$= (V_2 + \sqrt{160 + 1.25 V_2^2})^2 + 0.5 V_2^2$$

or

$$2 V_2 \sqrt{160 + 1.25 V_2^2} = 98 - 2.75 V_2^2 \quad (7)$$

which, upon squaring both sides, can be written as

$$V_2^4 - 460\, V_2^2 + 3748 = 0$$

By using the quadratic formula we can solve for V_2^2 to obtain either $V_2^2 = 452$ or $V_2^2 = 8.30$. Thus, either $V_2 = 21.3$ ft/s or $V_2 = 2.88$ ft/s. The value $V_2 = 21.3$ ft/s is not a root of the original equations. It is an extra root introduced by squaring Eq. 7, which with $V_2 = 21.3$ becomes "$1140 = -1140$." Thus, $V_2 = 2.88$ ft/s and from Eq. 5, $V_1 = 15.9$ ft/s. The corresponding flowrates are

$$Q_1 = A_1 V_1 = \frac{\pi}{4} D_1^2 V_1 = \frac{\pi}{4} (1 \text{ ft})^2 (15.9 \text{ ft/s})$$
$$= 12.5 \text{ ft}^3\text{/s from } A \quad \textbf{(Ans)}$$

$$Q_2 = A_2 V_2 = \frac{\pi}{4} D_2^2 V_2 = \frac{\pi}{4} (1 \text{ ft})^2 (2.88 \text{ ft/s}) \quad \textbf{(Ans)}$$
$$= 2.26 \text{ ft}^3\text{/s into } B$$

and

$$Q_3 = Q_1 - Q_2 = (12.5 - 2.26) \text{ ft}^3\text{/s}$$
$$= 10.2 \text{ ft}^3\text{/s into } C \quad \textbf{(Ans)}$$

Note the slight differences in the governing equations depending on the direction of the flow in pipe (2)—compare Eqs. 1, 2, and 3 with Eqs. 4, 5, and 6.

If the friction factors were not given, a trial-and-error procedure similar to that needed for Type II problems (see **Section 8.5.1**) would be required.

The ultimate in multiple pipe systems is a *network* of pipes such as that shown in Fig. 8.38. Networks like these often occur in city water distribution systems and other systems that may have multiple "inlets" and "outlets." The direction of flow in the various pipes is by no means obvious—in fact, it may vary in time, depending on how the system is used from time to time.

The solution for pipe network problems is often carried out by use of node and loop equations similar in many ways to that done in electrical circuits. For example, the continuity equation requires that for each *node* (the junction of two or more pipes) the net flowrate is zero.

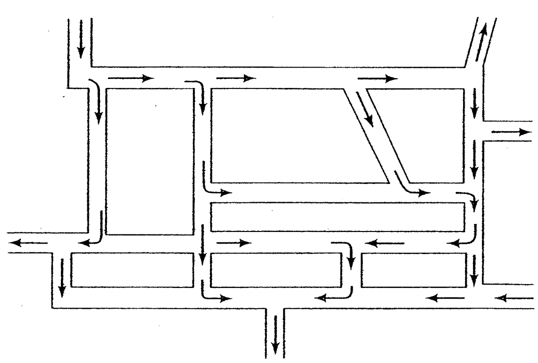


FIGURE 8.38 A general pipe network.