Hydraulic Design of Large-Diameter Pipes

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Abstract: The Hazen–Williams formula is frequently used for the design of large-diameter pipes, without regard for its limited range of applicability. This practice can have very detrimental effects on pipe design, and could potentially lead to litigation. Available evidence shows that the application of the formula is accurate only if the operation of the pipe is located within the transition or smooth, turbulent-flow regimes. Most working ranges for water-supply pipes usually fall outside such conditions. This paper presents an analysis which highlights the potential implications of current use of the Hazen–Williams formula for the design of large-diameter pipe systems.

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Introduction

Worldwide population growth has brought along a clear need to increase the capacity of water-supply and sewerage systems. Pipes having large diameters are commonly found in modern water-distribution networks located in major metropolitan areas. Consequently, the misuse of any design formulation to estimate flow resistance in large pipes can lead to serious practical drawbacks. More specifically, those systems might not be able to meet the demand they were designed for, thus reducing their useful life.

The Hazen–Williams formula, which dates back to the early 1900s, has traditionally been regarded as a general, simple tool to compute head losses in pipes carrying water (Chen 1992). The equation includes a conveyance coefficient usually assumed to be constant for a certain pipe-wall material, regardless of the pipe size or the flow range. This fact can result in deficient designs under a wide set of conditions. Unfortunately, the formula is currently being used outside of its actual range of validity throughout the United States and worldwide. Furthermore, in many cities and some counties throughout the States, any computation dealing with water flow in pipes is simply rejected if the Hazen–Williams formula is not used, without any regard for its intrinsic limitations (Diskin 1960; Jain et al. 1978; Liou 1998; Christensen 2000; Locher 2000; Swamee 2000).

In this paper, an analysis illustrating the implications of current use of available resistance formulations is presented. The case study that motivated the analysis refers to a large water-supply system for a major metropolitan area, originally designed with the help of the Hazen–Williams equation. After only 5 years in service, it was found that the network was inadequate in delivering the amount of water it had been designed for. The authors were asked to analyze why the recently built pipeline could not meet that water demand. The analysis consisted of: (1) reviewing the literature concerning issues such as range of validity and accuracy of the Hazen–Williams formula; (2) revisiting the original dataset employed by Hazen and Williams to develop the equation; and (3) using pressure-head and velocity measurements taken independently by two consulting engineering firms to estimate flow resistance (i.e., resistance coefficients) in the pipeline system in question.

In what follows, the analysis inspired by the case study is presented, followed by a discussion of the results, and the conclusions.

Preliminary Theoretical Considerations and Analysis

There are a number of equations commonly used to estimate flow resistance in channels and pipes. They are:

- Manning equation

$$U=\frac{K_n R^{2/3} S^{1/2}}{n} \tag{1}$$

- Dimensionally homogeneous Manning formula (Yen 1992b)

$$U=\frac{g^{1/2} R^{2/3} S^{1/2}}{n_g} \tag{2}$$

- Chezy equation

$$U=CR^{1/2}S^{1/2} \tag{3}$$

- Darcy–Weisbach equation

$$U=\left(\frac{2g}{f}\right)^{1/2} D^{1/2}\left(\frac{h_f}{L}\right)^{1/2} \tag{4}$$

- Hazen–Williams equation

$$U=K_{\mathrm{HW}} C_{\mathrm{HW}} R^{0.63} S^{0.54} \tag{5}$$

where U indicates the cross-sectional averaged velocity; R=hydraulic radius; S=slope of the energy grade line; D=pipe diameter; L=length of the reach of the pipe; h_f=energy loss in the pipe reach (expressed as per unit weight of fluid); g=acceleration of gravity; K_n and K_{HW}=unit conversion factors;

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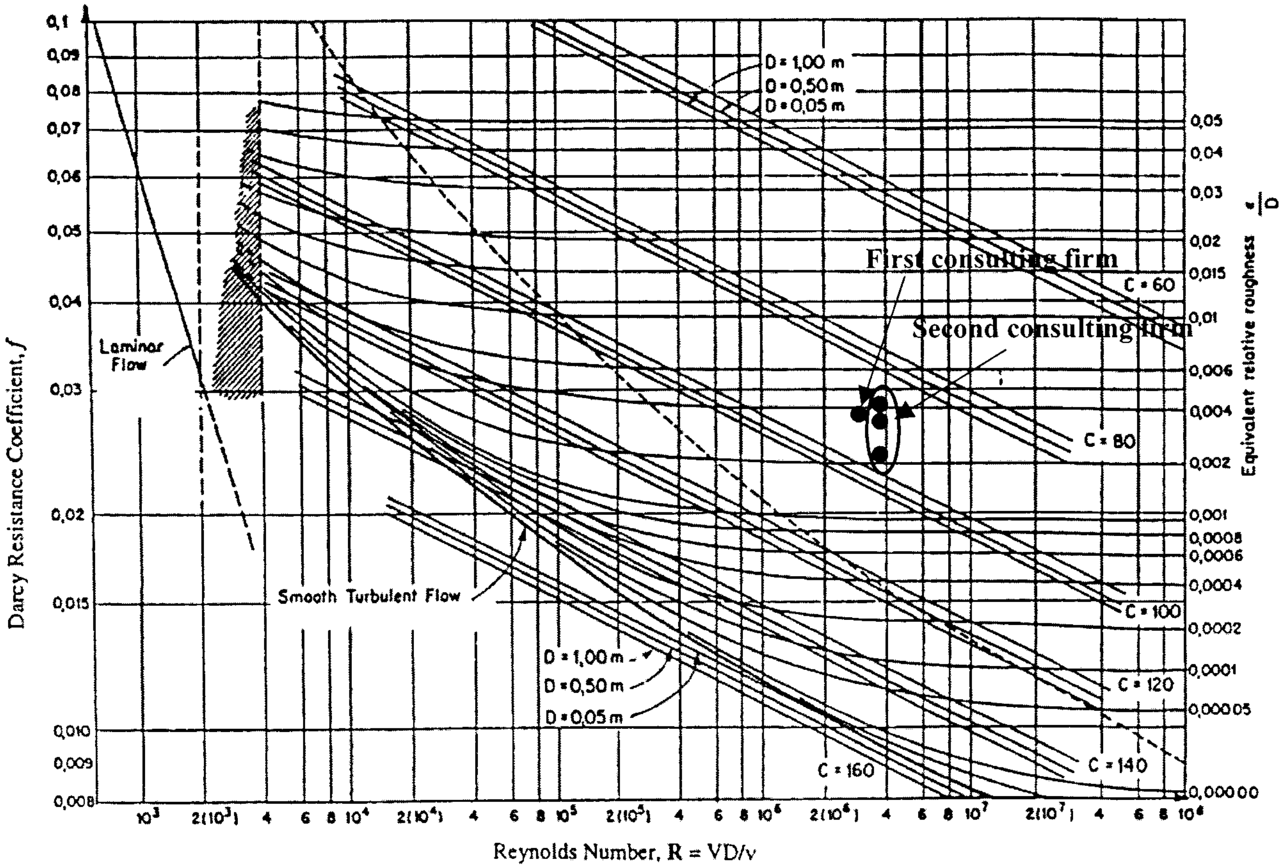

Fig. 1. Predictions obtained with the Hazen–Williams formula plotted in the Moody diagram [adapted from Diskin (1960)]. Operational points of the pipes pertaining to the case study (measurements by the two consulting engineering firms).

and n, n_g, C, f, and C_{HW} refer to the resistance/conveyance coefficients. K_n is equal to 1 $\mathrm{m}^{1/2}$/s (International System, SI) or to 1.486 $\mathrm{ft}^{1/3}\,\mathrm{m}^{1/6}$/s if English units are employed (Yen 1992b). For K_{HW}, values of 0.849 and 1.318 are used in each of the above systems, respectively (Jeppson 1977). Hazen and Williams evaluated the factor 1.318 as a means of keeping $C_{\mathrm{HW}}=C$, for a slope of 10^{-3} and a hydraulic radius of 0.305 m (1 ft) (Williams and Hazen 1920). A considerable number of papers has been devoted to the determination of values for C_{HW} pertaining to pipes made of different materials, including polyethylene pipes (Moghazi 1998). Also, several papers have addressed the optimization of the design of pipe networks, either using the Hazen–Williams formula [e.g., Datta and Sridharan (1994); and Niranjan Reddy et al. (1996)], or modified formulations [Elimam et al. (1989); and Charalambous and Elimam (1990)].

It has been well known since the beginning of the last century (King et al. 1948), that the Darcy–Weisbach and Manning formulas can be both used for computations in either open-channel or pipe, fully rough turbulent flow. Thus it is possible to determine backwater profiles in open channels using the Darcy–Weisbach equation or, conversely, to compute energy losses in pipes employing Manning's equation, provided the convenient equivalence $D=4R$ is considered, and reliable estimates for the resistance coefficients are at hand (Yen 1992a, 2002). The Chezy equation can also be employed in either case. However, the Hazen–Williams formula has quite a restricted range of application that limits its use.

In their classic monograph, Williams and Hazen (1920) stated that the exponents of their equation depended on pipe diameter and slope. Vennard (1958) highlighted that one of the disadvantages of the equation was related to "the impossibility of applying it to all fluids under all conditions." Nevertheless, Diskin (1960) was apparently *the first* to strongly acknowledge the limitations associated with the formula and to undertake a rather careful analysis of its range of validity. Diskin first rearranged the Hazen–Williams equation in the shape of the Darcy–Weisbach formula, in a similar way as previously done by Vennard (1958). Thus he was able to obtain a power relation between f, C_{HW}, D, and R (the pipe Reynolds number, $\mathsf{R}=UD/\nu$; ν is the water kinematic viscosity), that reads for D in meters

$$f=\frac{0.2004(100/C_{\mathrm{HW}})^{1.852}}{D^{0.019}}\frac{1}{\mathsf{R}^{0.148}} \tag{6}$$

In Eq. (6), the value of water viscosity at 15°C was employed. This expression demonstrates that C_{HW} is not a constant for a certain wall condition, as is interpreted in numerous engineering manuals (Liou 1998). Instead, it depends on the flow condition, the pipe diameter and relative roughness, and the water temperature, through the kinematic viscosity, ν.

Based on this result, Diskin plotted Eq. (6) for different values of C_{HW} and D into the Moody diagram (Fig. 1). Using this plot, he was able to conclude that the formula is applicable "in part of the transition zone." Diskin determined the ranges of Reynolds numbers for which the original lines of the Moody diagram are parallel to the lines representing the predictions of the Hazen–Williams equation. Those limiting Reynolds numbers increase

Table 1. Measurements Performed by Two Engineering Firms (Values Computed by Authors)

(a)
First line corresponds to the main pipe; second line pertains to another pipe in the system with another discharge.

Pipe diameter (m)	Area (m^2)	Velocity measured (m/s)	Losses measured (m)	Reach length (m)	Slope of energy line	C_{HW}	f	n	n_g ($m^{1/6}$)	C_f	C ($m^{1/2}$/s)	R
2.286	4.104	1.076	9.604	13,692.53	0.00070	91	0.027	0.017	0.053	0.0034	54	2,450,000
1.829	2.627	0.960	1.884	3,213.506	0.00059	103	0.023	0.015	0.047	0.0029	59	1,750,000

(b)
All measurements pertain to the main pipes.

Pipe reach	Losses measured (m)	Reach length (m)	Slope of energy line	C_{HW}	f	n	n_g ($m^{1/6}$)	C_f	C ($m^{1/2}$/s)
1–2	6.748	3,959.352	0.00170	87	0.028	0.017	0.054	0.0035	53
2–3	7.187	5,179.771	0.00139	97	0.023	0.015	0.049	0.0028	59
3–4	7.139	4,560.113	0.00157	91	0.026	0.016	0.052	0.0032	55

Note: D (m) = 2.286, Q (m^3/s) = 6.795, U (m/s) = 1.655, Velocity head (m) = 0.140, Reynolds number = 3.8×10^6.

with decreasing relative roughness. Diskin also stated that the application of the formula in the appropriate range should result in values of C_{HW} between 100 and 160.

Barlow and Markland (1975) and Jain et al. (1978) presented similar analyses regarding the accuracy of the Hazen–Williams equation. In both papers, the Hazen–Williams expression was recast into a Darcy–Weisbach-type formula. Thus equations that give f in terms of C_{HW} and other parameters, very similar to Eq. (6), were obtained. Jain et al. then introduced values of C_{HW} in their resulting formula, and the outcoming values of f were compared to counterparts obtained with an accurate, explicit, Colebrook–White-type equation, previously developed by Jain (1976). This was done for different values of slope and hydraulic radius. Considering the Colebrook–White-type expression as exact, errors of up to ±40% were computed. Jain et al. concluded that two independent sources of error affect the Hazen–Williams equation, namely, the change in the value of the factor 1.318 in terms of R and S and the change of C_{HW} with flow condition and pipe diameter. These authors finally presented a modified Hazen–Williams formula.

Recently, Liou (1998) derived, through similar procedures as those employed by Vennard (1958); Diskin (1960); Barlow and Markland (1975); Jeppson (1977); and Jain et al. (1978), the following expression:

$$C_{HW}=14.07f^{-0.54}\mathsf{R}^{-0.08}D^{-0.01}\nu^{-0.08} \tag{7}$$

Liou plotted C_{HW} in terms of R and ε/D (where ε indicates the equivalent roughness of the pipe) for different pipe diameters. In doing so, he used Eq. (7), he adopted a reasonable value for ν, he held ε constant, and computed f from the Colebrook–White formula. Included in Liou's graphs, there are some of the data points, pertaining to new cast-iron pipes, in which Hazen and Williams based their expression. Liou concluded that those measurements "cover only a portion of the transition zone," in agreement with the statement made much earlier by Diskin. He computed the errors in the prediction of the slope of the energy grade line when the Hazen–Williams formula is used instead of the Darcy–Weisbach equation. Liou found that those errors are relatively small if the Hazen–Williams formula is applied within the corresponding range (transition regime), but that they can attain values up to ±40% when used outside of the appropriate range. Finally, he recommended avoiding the use of the Hazen–Williams formula.

Very recently, Christensen (2000) used a regression of Nikuradse's formula for f in the smooth regime (valid between R $=10^5$ and 10^8), i.e., $f=0.1079/\mathsf{R}^{0.16}$, and obtained an equation with the same form of the Hazen–Williams equation. He had followed the same analysis in another discussion (Christensen 1984), together with a derivation of Manning's formula. Christensen thus suggested a diagram with zones of application of the Hazen–Williams and Manning formulas. According to this plot, there would be a minimum value of D/ε below which the formula does not apply. It becomes clear that if the Hazen–Williams equation is accepted to be also valid in part of the transition range, the limiting value of $D/\varepsilon=1{,}441$ at a Reynolds number of 10^5, put forward by Christensen, decreases. Finally, Swamee (2000) pointed out some theoretical inconsistencies related to the Hazen–Williams formula and recalled that C_{HW} has dimensions of length to the 0.37 power over time.

A general frame of reference for all resistance equations can be obtained following Yen (1992a, 2002). Since the cross-sectional averaged velocity has the same meaning in Eqs. (1)–(4) and because all the formulas represent the same resistive phenomenon, it follows that

$$\sqrt{\frac{8}{f}}=\frac{C}{\sqrt{g}}=\frac{K_n}{\sqrt{g}}\frac{R^{1/6}}{n}=\frac{R^{1/6}}{n_g}=\frac{U}{\sqrt{gRS}} \tag{8}$$

The denominator in the last right-hand side is called the shear velocity, U_*, i.e.

$$U_*=\sqrt{gRS} \tag{9}$$

Eq. (9) can help in determining the so-called friction coefficient, C_f, as follows:

$$C_f=\left(\frac{U_*}{U}\right)^2 \tag{10}$$

According to Eq. (10) and to the definition of the resistance coefficients for each equation, it is possible to write

$$C_f=\frac{f}{8}=\frac{g}{C^2}=\frac{g}{K_n^2}\frac{n^2}{R^{1/3}}=\frac{n_g^2}{R^{1/3}} \tag{11}$$

Case Study

The case study corresponds to a water-supply system located in a large metropolitan area, which due to potential legal implications shall remain anonymous. The network, which serves a population of approximately one million, consists of pipes of different sizes and materials, having internal diameters as large as 2.29 m (90 in.). The main pipes are made of concrete. For the hydraulic design of the main pipes, the Hazen–Williams formula was employed and $C_{HW}=120$ was adopted. This value is commonly accepted as a conservative value for good masonry aqueducts; see Williams and Hazen (1920).

After 5 years in operation, a set of hydraulic tests was performed on several pipes of the system by a consulting engineering firm. Pressure heads at the ends of the pipes and flow velocities were measured. The Hazen–Williams conveyance coefficient was later computed from those measurements. *The results showed that the calculated C_{HW} was significantly lower than the one used for the design, ranging from 85 to 95 for the 2.29 m ϕ pipes.* Williams and Hazen (1920) presented similar low values for the coefficient in their classic manual; however, they pertain to tuberculated cast-iron pipes. A cursory review would suggest that these lower values of C_{HW} relate to larger equivalent roughness heights than first anticipated in the design. If these measurements were correct, an increase in the demand for water in upcoming years would not be satisfied as planned, because the resistance to the flow in these main pipes (which are of primary importance to any water-distribution system) would be much higher than expected.

These results spread disbelief among the technical staff of the water commission responsible for the operation of the network. Having these results and knowing the limited range of applicability of the Hazen–Williams formula, it was not clear if this situation was based on a problem with the formula or it was due to a real increase in the roughness of the pipes. The water commission's initial efforts were focused on potential errors of the measurements. Those observations were likely to have erroneously included local losses in the computation of the coefficient, such as those associated with partially closed valves or with the presence of bends. Another source of inaccuracies could have stemmed from the placement of the velocity-measurement cross sections at less than ideal locations for accurately measuring flow velocity distributions.

To verify that measurement error was not the cause of the low C_{HW} values, a separate consulting engineering firm was asked to undertake a second set of tests. For such a set of observations, special attention was given to the above "hidden" losses and measurement location issues. The measurements focused on different reaches of just the 2.29 m diameter pipe. The tests and related computations confirmed the values of C_{HW} reported by the first engineering firm. Overall, these careful analyses concluded that local losses could modify C_{HW} in less than three units. Therefore the ranges for C_{HW} originally reported were almost unaffected. Tables 1(a) and 1(b) depict some of the measurements performed by the two consulting firms in the main and other pipes, and include additional calculations made by the authors.

After verification of the above results, the authors were asked to analyze the possible causes for the observed behavior. The analysis undertaken led to some interesting conclusions, which are detailed in the following paragraphs.

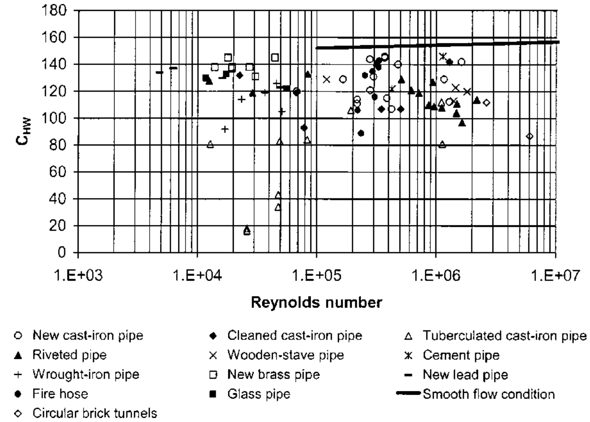

Fig. 2. Compilation of average values for the Hazen–Williams resistance coefficient as a function of Reynolds number

Analysis and Discussion

Theoretical Analysis

As a step towards the analysis of the case study, the original dataset used by Hazen and Williams to obtain their formula was first reviewed (Williams and Hazen 1920). These data include measurements for pipe *and open-channel flows*. The largest pipe diameter compiled by Hazen and Williams pertains to a sewer in Milwaukee, of 3.66 m (144 in.), with a C_{HW} ranging from 80 to 95. Most of the pipes studied have diameters smaller than 1.78 m (70 in.). In fact, almost 74% of the compiled diameters are below 0.5 m, 82% are below 1 m, and 92% are below 1.5 m. In turn, 71% of the cases analyzed in the Williams and Hazen manual relate to Reynolds numbers below 5×10^5, while 80% relate to Reynolds numbers below 10^6. In the computation of the Reynolds numbers, a value of water kinematic viscosity corresponding to 20°C, $\nu=1.005\times10^{-6}\,\mathrm{m^2/s}$, was employed. Interestingly, the majority of the C_{HW} values below Diskin's lower limit of 100 pertains to tuberculated cast-iron pipes. Fig. 2 relates *average* values of C_{HW} to pipe Reynolds numbers in a similar way as presented by Liou (1998) (only new cast-iron pipes were used in Liou's paper), where it is seen that the analyzed Reynolds numbers range approximately from 10^4 to 2×10^6. Additionally, Fig. 2 includes a $C_{HW}-\mathsf{R}$ curve for smooth flow conditions, obtained with the help of the regression to Nikuradse's formula (mentioned above) and Eq. (7). The influence of the diameter was ignored in the use of Eq. (7), by virtue of the very weak dependence of C_{HW} on D through the exponent -0.01 (this is accurate to within 3% for a range of diameters from 0.1 to 10 m). In Fig. 2, the transition regime exists below this smooth flow curve. A lower limit for the transition region could be given by curves like those presented by Liou (1998), which are a function of the relative roughness and the pipe diameter, for a given water temperature. Most of the measurements used by Hazen and Williams appear to pertain to part of the transitional turbulent regime, which corroborates the previously mentioned "theoretical" assertions. Also, the proximity of the points to the smooth-regime curve would support the validity of the equation for that range as well.

Large pipes (>2 m diameter) show an interesting behavior. Because of their size, they are related to high Reynolds numbers. Hence they are likely to present operational conditions in the fully rough turbulent regime. However, because of the high value of the diameter, ε/D is very small for a certain ε. Therefore the chances of having working conditions in the transition turbulent regime increase. Thus the larger the diameter, the higher are the chances of finding appropriate conditions for the application of the Hazen–Williams formula, for constant equivalent roughness and Reynolds number (see Christensen 2000).

Since the problem under analysis could "a priori" be attributed to a large equivalent roughness, different expressions for the computation of ε were examined. Several explicit relations have been proposed for pipes that link f to ε. For example, Churchill (1973) and Barr (1972, 1977) have proposed

$$f=\frac{0.25}{\left[-\log\left(\frac{\varepsilon}{14.8R}+\frac{5.76}{(4\mathsf{R}_{\mathsf{R}})^{0.9}}\right)\right]^2} \tag{12}$$

with R_{R} being a Reynolds number based on the hydraulic radius. Swamee and Jain (1976), in turn, have suggested virtually the same relation (Yen 1992a, 2002)

$$f=\frac{0.25}{\left[-\log\left(\frac{\varepsilon}{3.7D}+\frac{5.74}{\mathsf{R}^{0.9}}\right)\right]^2} \tag{13}$$

which gives errors within 1% when compared to the Colebrook–White formula for $10^{-6}\leqslant\varepsilon/D\leqslant10^{-2}$ and $5\times10^3\leqslant\mathsf{R}\leqslant10^8$. Since Manning's n can be related to Darcy–Weisbach's f through Eq. (11) as follows:

$$n=\frac{K_n}{\sqrt{g}}\frac{R^{1/6}}{\sqrt{8}}\sqrt{f} \tag{14}$$

it is possible to obtain a relation between n and ε using Eq. (12) or (13). Using Eq. (12) yields

$$n=\frac{K_n}{\sqrt{g}}\frac{0.1764R^{1/6}}{\left[-\log\left(\frac{\varepsilon}{14.8R}+\frac{5.76}{(4\mathsf{R}_{\mathsf{R}})^{0.9}}\right)\right]} \tag{15}$$

This expression can be compared to other available formulas. One such formula is the well-known Strickler-type relation shown below, usually applied to open-channel flows but obtained also from pipe data (Strickler 1923; Ackers 1961; Chow 1988; Yen 1992a):

$$n=\frac{\varepsilon^{1/6}}{C_n} \tag{16}$$

Different authors have provided diverse values for C_n in Eq. (16) (Yen 1992a). Chien and Wan (1999) explained that C_n is a function of R/ε, while Strickler, in his original work (1923), suggested $C_n=21.1$.

The comparison between Eqs. (15) and (16) is shown in Fig. 3, in which a value of 26 has been used for C_n (a value of 26.42 was first proposed by Williamson in 1951). A very large value for the pipe Reynolds number has been adopted in the computations. It can be seen that both formulas give relatively close predictions up to a Manning's n value of about 0.015. After that, they separate due to the highly nonlinear behavior of the Strickler formula. A close inspection of the curves indicates that, however, the local error [considering Eq. (15) as exact] can be relatively high, depending on the value of the hydraulic radius. In order to calculate ε the authors relied on Eq. (12).

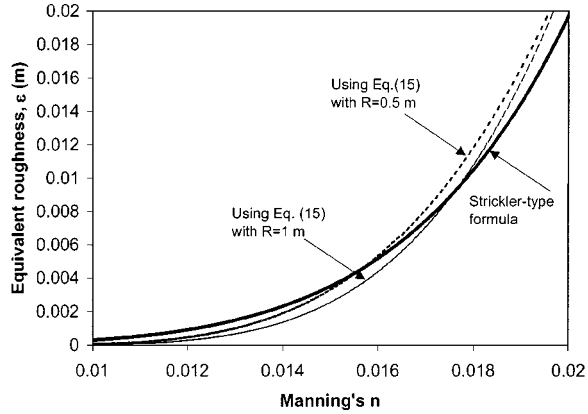

Fig. 3. Comparison of predictions for the equivalent roughness of pipes as a function of Manning's n, obtained with Eqs. (15) and (16)

Sensitivity Analysis

It is interesting to analyze the sensitivity of the different resistance coefficients to detect, in a given pipe reach, the presence of combined changes in the head loss and in the velocity. Such head-loss and velocity changes can be interpreted as errors in the variables that both consulting firms measured in the 2.29 m ϕ pipes. To that end, relative errors were calculated. Variables related to length were assumed as having been measured with negligible error. For any resistance coefficient C_R, the following is true:

$$\frac{dC_R}{C_R}=\frac{\partial C_R}{\partial h_f}dh_f\frac{1}{C_R}+\frac{\partial C_R}{\partial U}dU\frac{1}{C_R} \tag{17}$$

For example, applying Eq. (17) to Manning's equation

$$n=\frac{h_f^{1/2}}{U}\frac{K_nR^{2/3}}{L^{1/2}} \tag{18}$$

yields

$$\frac{dn}{n}=\frac{1}{2}\frac{dh_f}{h_f}-\frac{dU}{U} \tag{19}$$

Repeating this simple procedure with the other resistance equations results in the following:

$$\frac{df}{f}=\frac{dh_f}{h_f}-2\frac{dU}{U} \tag{20}$$

$$\frac{dC_{\mathrm{HW}}}{C_{\mathrm{HW}}}=\frac{dU}{U}-0.54\frac{dh_f}{h_f} \tag{21}$$

$$\frac{dC}{C}=\frac{dU}{U}-\frac{1}{2}\frac{dh_f}{h_f} \tag{22}$$

$$\frac{dn_g}{n_g}=\frac{1}{2}\frac{dh_f}{h_f}-\frac{dU}{U} \tag{23}$$

It can be seen from the above simple equations that the value of the relative change in the resistance coefficients is linearly determined by relative errors in the head-loss and in the velocity measurements, with the corresponding sign. Relative errors thus plot as a plane in the three-dimensional space in terms of the relative errors in head loss and velocity.

Since the head loss is usually calculated from the differences in pressure heads and in elevation within the ends of the pipes (through the energy balance), and since a negligible error in the measurement of the distances was assumed, it holds that

Table 2. Computation of Shear Velocity, Viscous Sublayer Thickness and Equivalent Roughness for Main Pipes

Pipe reach	f	Reach length (m)	Slope of energy line	U_* (m/s)	δ_v (m)	ε (m)	ε/δ_v
1–2	0.028	3,959.352	0.00170	0.098	0.00012	0.009	72
2–3	0.023	5,179.771	0.00139	0.088	0.00013	0.004	31
3–4	0.026	4,560.113	0.00157	0.094	0.00012	0.007	54

$$\frac{dh_f}{h_f} \approx \frac{d(\Delta p/\gamma)}{h_f} \tag{24}$$

The above analysis was applied to the data obtained by the two consulting firms in the network of pipes.

Discussion of the Case Study

The first question arising from the analysis of the observations was related to the validity of the application of the Hazen–Williams equation to the design of the pipe network. To answer this question, the values of C_{HW}, n, C, f, and n_g were first determined from the measurements, using Eqs. (1)–(5) [see Tables 1(a) and 1(b)]. In particular, f and the Reynolds number were employed to obtain the operational points of the pipes during the measurements, using the Moody diagram (see Fig. 1). For these measurements, it is clearly seen in Fig. 1 that the high Reynolds numbers and the f values involved place the flow in the fully rough turbulent regime. As it has been previously shown, this falls outside of the valid flow regime for the Hazen–Williams formula to be accurate. Notice from Fig. 1 that the corresponding C_{HW} value pertains to the range 80–100, as calculated by both firms; however, *these values are meaningless, as the Hazen–Williams equation is not applicable in this range of flow conditions.* This is consistent with Diskin's conclusion that the Hazen–Williams formula is not to be used if C_{HW} is lower than 100.

Values were computed for the shear velocity, viscous sublayer thickness, and the ratio of equivalent roughness to viscous sublayer thickness for the 2.29 m ϕ pipe, based on the measurements performed by the second engineering firm (Table 2). Knowing S and R, the shear velocity was computed by definition. In turn, the equivalent roughness values were obtained from Eq. (12) [where f was taken from Tables 1(a) and 1(b)], and the viscous sublayer thickness was computed from (Rouse 1978)

$$\delta_v = 11.6\frac{\nu}{U_*} \tag{25}$$

As seen in Table 2, the calculated value of ε oscillated between 4×10^{-3} and 9×10^{-3} m, with an average value around 6 mm. These values are fairly large for a cement-mortar pipe, which typically presents an equivalent roughness between 5×10^{-4} and 3.3×10^{-3} m (Yen 1992a).

Even though the Hazen–Williams formula should have not been applied to estimate flow capacity of the 2.29 m diameter pipes, the analysis of the data was pursued further to shed some light on the interpretation of the measurements. The idea was to see whether or not other formulations showed larger values of the flow resistance with respect to the expected conditions. As a starting point, a value of Manning's n of 0.014 was assumed to be the expected value (in a statistical sense) from the measurements. This value is widely accepted for cement-mortar pipes (Yen 1992a). Since the system was only 5 years old, it was reasonable to assume that the value of the resistance coefficient would be close to the one for a new pipe (Hudson 1966). Using the relation between f and n [Eq. (14)] for a diameter of 2.29 m, it was possible to obtain $f=0.0185$, another acceptable value. Employing the set of relations in Eq. (11), expected values for the other coefficients and for the friction coefficient were calculated as follows: $n_g=0.0438\,\mathrm{m}^{1/6}$, $C=65\,\mathrm{m}^{1/2}/\mathrm{s}$, and $C_f=0.0023$. Incidentally, the above value for f can be employed to obtain an estimate of the error in the design of the pipes. In fact, if a value of $C_{\mathrm{HW}}=120$ is replaced in Eq. (7) together with the Reynolds number (3.8×10^6), it is possible to obtain $f=0.0155$, with a difference from 0.0185 of about 16%.

Using Eqs. (19)–(23), a sensitivity analysis of the possible variation of the resistance coefficients in terms of the errors in the head loss and the flow velocity can be performed. For example, for a relative error of $\pm2\%$ in the measurement of the head loss and $\pm3\%$ in that of velocity, it can be obtained from Eq. (19) that $|dn|\leqslant0.04\,n$. Then, taking the value of 0.014 for the Manning's resistance coefficient, $|dn|\leqslant5.6\times10^{-4}$. This means that n can range from 0.0134 to 0.0146. For other head-loss and velocity errors, Table 3 presents the results of the computations. Similar ranges for the other resistance coefficients can also be found (see Table 3). Any expectation for the final value of the resistance coefficient has to be expressed in terms of a percentage of error in the measurement of the head loss and velocity, assuming the remaining variables are measured with negligible error. Thus Table 3 gives the ranges of plausible variation for the resistance coefficients.

If the numbers in Tables 1(a) and 1(b) are compared with the ranges depicted in Table 3, it can be seen that all the values, independent of the resistance equation employed, indicate a flow resistance that is larger than the expected one. This is based on the fact that the values of the coefficients in Tables 1(a) and 1(b) are larger (f,n,n_g,C_f) or smaller (C) than the largest/smallest corresponding value in each of the ranges presented in Table 3. This additional resistance could be attributed to an existing larger equivalent roughness. Notice that the calculated roughness (Table

Table 3. Ranges of Expected Values for Resistance Coeffecients in Terms of Errors in Measurements

$(dh_f)/h_f$	$(dU)/U$	$n_{\min}$	$n_{\max}$	$f_{\min}$	$f_{\max}$	$n_{g(\min)}$ $(\mathrm{m}^{1/6})$	$n_{g(\max)}$ $(\mathrm{m}^{1/6})$	$C_{\min}$ $(\mathrm{m}^{1/2}/\mathrm{s})$	$C_{\max}$ $(\mathrm{m}^{1/2}/\mathrm{s})$	$C_{f(\min)}$	$C_{f(\max)}$
±0.02	±0.03	0.01344	0.0146	0.0171	0.0200	0.0421	0.0456	62	68	0.0021	0.0025
±0.04	±0.06	0.0129	0.0151	0.0156	0.0215	0.0403	0.0474	60	70	0.0019	0.0027
±0.06	±0.09	0.0123	0.0157	0.0141	0.0230	0.0386	0.0491	57	73	0.0018	0.0029

2) is much higher (about 60 times higher) than the value of the viscous sublayer thickness.

It is noteworthy that this result is a physical fact: the equivalent roughness is indeed large. This is not the result of an erroneous application of the Hazen–Williams equation. The problem consists in that, *after the application of the Hazen–Williams formula* outside of its range of validity, the conclusions that can be drawn from it are meaningless. In other not so fortunate cases, difficulties in meeting future water-supply demands could be the result of improper application of the Hazen–Williams equation.

Interestingly, an inspection of the pipe system, conducted by divers, revealed that small scales having heights on the order of several millimeters covered the walls of the pipes. In spite of the fact that several other factors might be playing a role in the results, the presence of these scales could very well be the cause for the observed increase in flow resistance and the associated large values of the equivalent roughness ε (Table 2).

Summary and Conclusions

Several issues surrounding the Hazen–Williams formula have been discussed in light of a case study related to a large water-distribution system.

An analysis of the valid flow range of the Hazen–Williams formula, via inspection of the original dataset, has been presented. The analysis confirmed that this range lies in the smooth and part of the transition turbulent regimes, as stated previously by several authors with the help of other means. The formula was developed from data for pipes with diameters ranging up to 3.66 m, but most of the data were collected for pipes with diameters below 1.8 m. The Reynolds numbers involved in the determination of the formula spanned from approximately 10^4 to 2×10^6.

A procedure for the determination of the sensitivity of the various resistance coefficients to head-loss and velocity measurement errors was revisited. The analysis assumed that the remaining variables were measured with null error. This analysis provides a basis for the determination of expected ranges for those coefficients when measuring in the field or in the laboratory.

Values for the equivalent roughness, computed from explicit expressions, were compared to predictions obtained using Strickler-type formulas. The local error in the values of equivalent roughness given by Strickler-type formulas depends strongly on the hydraulic radius for a certain value of Manning's n.

The case study, related to a large-diameter pipe system, was used as an example of the current misuse of the Hazen–Williams equation outside of its proper range of validity. The operation flow regime was in the fully rough turbulent regime. When demand increases, the continuous operation will generate a larger equivalent roughness and, thus, the working points will be farther away from the transition regime. Therefore any future use of the Hazen–Williams formula for verification of the flow resistance in that network would also be meaningless. In this regard, the authors strongly recommend using the Darcy–Weisbach equation, which includes all flow regimes.

Finally, it is important to point out that the indiscriminate application of the Hazen–Williams formula either in the design or verification of water-supply systems is far from a simple academic problem. It may lead to serious practical and conceptual implications in otherwise straightforward computations.

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Notation

The following symbols are used in this paper:

C = conveyance coefficient in Chezy equation;
C_f = friction coefficient;
C_{HW} = conveyance coefficient in Hazen–Williams equation;
C_n = coefficient in Strickler-type relations;
C_R = generic resistance/conveyance coefficient;
D = pipe diameter;
f = resistance coefficient in Darcy–Weisbach formula;
g = acceleration of gravity;
h_f = energy loss in the pipe reach (expressed per unit of weight);
K_n and K_{HW} = unit conversion factors for the Manning and Hazen–Williams formulas, respectively;
L = length of the reach of the pipe;
n, and n_g = resistance coefficients of the Manning and dimensionally homogeneous Manning formulas, respectively;
Q = discharge of a given pipe;
R = pipe Reynolds number, based on the diameter;
R_{R} = pipe Reynolds number, based on the hydraulic radius;
R = hydraulic radius;
S = slope of the energy grade line;
U = cross-sectional averaged velocity;
U_* = shear velocity;
Δp = pressure difference between the ends of the pipe;
δ_v = viscous sublayer thickness;
ε = equivalent roughness of pipe;
ν = kinematic viscosity of water; and
ρ = density of water.

References

Ackers, P. (1961). ''The hydraulic resistance of drainage conduits.'' *Proc. Inst. Civ. Eng. London*, 19, 307–336.

Barlow, J. F., and Markland, E. (1975). ''Converting the Hazen-Williams equation to the Colebrook function.'' *Water Power Dam Constr.*, 9, 331–334.

Barr, D. I. H. (1972). ''New forms of equations for the correlation of pipe resistance data.'' *Proc. Inst. Civ. Eng. London*, 53(2), 383–390.

Barr, D. I. H. (1977). ''Discussion on 'Accurate explicit equation for friction factor,' by A. K. Jain.'' *J. Hydraul. Div., Am. Soc. Civ. Eng.*, 103(3), 334–337.

Charalambous, C., and Elimam, A. A. (1990). ''Heuristic design of sewer networks.'' *J. Environ. Eng.*, 116(6), 1181–1199.

Chen, C. L. (1992). ''Power law of flow resistance in open channels: Manning's formula revisited.'' *Channel flow resistance: Centennial of Manning's formula*, B. C. Yen, ed., Water Resources Publications, Littleton, Colo., 206–240.

Chien, N., and Wan, Z. (1999). *Mechanics of sediment transport*, ASCE Press, Reston, Va.

Chow, V. T. (1988). *Open-channel hydraulics*, McGraw-Hill, New York, classic reissue.

Christensen, B. A. (1984). ''Discussion on 'Flow velocities in pipelines,' by R. D. Pomeroy.'' *J. Hydraul. Eng.*, 110(10), 1510–1512.
Christensen, B. A. (2000). ''Discussion on 'Limitations and proper use of the Hazen-Williams equation,' by C. P. Liou.'' *J. Hydraul. Eng.*, 126(2), 167–168.
Churchill, S. W. (1973). ''Empirical expressions for the shear stress in turbulent flow in commercial pipe.'' *AIChE J.*, 19(2), 375–376.
Datta, R. S. N., and Sridharan, K. (1994). ''Parameter estimation in water-distribution systems by least-squares.'' *J. Water Resour. Plan. Manage. Div., Am. Soc. Civ. Eng.*, 120(4), 405–422.
Diskin, M. H. (1960). ''The limits of applicability of the Hazen-Williams formula.'' *Houille Blanche*, 6, 720–723.
Elimam, A. A., Charalambous, C., and Ghobrial, F. H. (1989). ''Optimum design of large sewer networks.'' *J. Environ. Eng.*, 115(6), 1171–1190.
Hudson, W. D. (1966). ''Studies of distribution system capacity in seven cities.'' *J. Am. Water Works Assoc.*, 58(2), 157–164.
Jain, A. K. (1976). ''Accurate explicit equation for friction factor.'' *J. Hydraul. Div., Am. Soc. Civ. Eng.*, 102(5), 674–677.
Jain, A. K., Mohan, D. M., and Khanna, P. (1978). ''Modified Hazen-Williams formula.'' *J. Environ. Eng. Div. (Am. Soc. Civ. Eng.)*, 104(1), 137–146.
Jeppson, R. W. (1977). *Analysis of flow in pipe networks*, Ann Arbor Science, Mich.
King, H. W., Wisler, C. O., and Woodburn, J. G. (1948). *Hydraulics*, Wiley, New York.
Liou, C. P. (1998). ''Limitations and proper use of the Hazen-Williams equation.'' *J. Hydraul. Eng.*, 124(9), 951–954.
Locher, F. A. (2000). ''Discussion on 'Limitations and proper use of the Hazen-Williams equation,' by C. P. Liou.'' *J. Hydraul. Eng.*, 126(2), 168–169.
Mogazhi, H. E. M. (1998). ''Estimating Hazen-Williams coefficient for polyethylene pipes.'' *J. Transp. Eng.*, 124(2), 197–199.
Niranjan Reddy, P. V., Sridharan, K., and Rao, P. V. (1996). ''WLS method for parameter estimation in water distribution networks.'' *J. Water Resour. Plan. Manage. Div., Am. Soc. Civ. Eng.*, 122(3), 157–164.
Rouse, H. (1978). *Elementary mechanics of fluids*, Dover Ed., Dover, New York.
Strickler, A. (1923). ''Contributions to the question of a velocity formula and roughness data for streams, channels and close pipelines.'' Translation by T. Roesgen and W. R. Brownlie, W. M. Keck Laboratory of Hydraulics and Applied Science, California Institute of Technology, 1981.
Swamee, P. K. (2000). ''Discussion on 'Limitations and proper use of the Hazen-Williams equation,' by C. P. Liou.'' *J. Hydraul. Eng.*, 126(2), 169–170.
Swamee, P. K., and Jain, A. K. (1976). ''Explicit equations for pipe-flow problems.'' *J. Hydraul. Div., Am. Soc. Civ. Eng.*, 102(5), 657–664.
Vennard, J. K. (1958). *Elementary fluid mechanics*, 3rd Ed., Wiley, New York.
Williams, G. S., and Hazen, A. (1920). *Hydraulic tables*, Wiley, Brooklyn, N.Y.
Williamson, J. (1951). ''The laws of flow in rough pipes. Strickler, Manning, Nikuradse and drag-velocity.'' *Houille Blanche*, 6(5), 738–748.
Yen, B. C. (1992a). ''Hydraulic resistance in open channels.'' *Channel flow resistance: Centennial of Manning's formula*, B. C. Yen, ed., Water Resources Publications, Littleton, Colo., 1–135.
Yen, B. C. (1992b). ''Dimensionally homogeneous Manning's formula.'' *J. Hydraul. Eng.*, 118(9), 1326–1332.
Yen, B. C. (2002). ''Open channel flow resistance.'' *J. Hydraul. Eng.*, 128(1), 20–39.