ECI 141: ENGINEERING HYDRAULICS

MID-TERM EXAMINATION

FALL 2005

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NAME ....

ANSWER ALL QUESTIONS.

For water, take $\rho = 10^3$ kg/m^3 (=1.94 slug/ft^3) and $\mu = 10^{-3}$ kg/m.s (=2.09 $\times$ 10^{-5} slug/ft.s).

QUESTION 1

A cast–iron horizontal pipe is used to carry water at a mean velocity of 6 ft/s.

The pipe has diameter D = 6 inches, length L = 200 ft and has roughness length ϵ= 0.0004 ft.

$D = 6 in. \cdot \frac{1 ft}{12 in} = 0.5 ft$

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Calculate:

1. the head loss,
2. the pressure drop,
3. the input power to the pump required to convey the water (assume pump efficiency of 70%).

$L = 200\ ft$

$\varepsilon = 0.0004 ft \rightarrow \frac{\varepsilon}{D} = \frac{0.0004\ ft}{0.5\ ft} = 8 \cdot 10^{-4}$

$V = 6\ \frac{ft}{s}$

$\mu = 2.09 \cdot 10^{-5}\ \frac{slug}{ft \cdot s}$ $\rho = 1.94\ \frac{slug}{ft^3}$

1.

$Re = \frac{\rho V D}{\mu} = \frac{(1.94)(6)(0.5)}{2.09 \cdot 10^{-5}} = 2.8 \cdot 10^5$ (turbulent)

$\varepsilon/D = 8 \cdot 10^{-4} = 0.0008$

use M-C with $\varepsilon/D = 0.0008$ and $Re = 2.8 \cdot 10^5$

$\rightarrow f \approx 0.0185$

$h_f = f \frac{L}{D} \frac{V^2}{2g} = (0.0185) \frac{(200)}{(0.5)} \frac{(6^2)}{(2 \cdot 32.2)} = 4.14\ ft \rightarrow \boxed{h_f = 4.14\ ft}$ +10

2.

$\frac{\Delta p}{\rho g} = h_f \rightarrow \Delta p = \rho g h_f = (1.94)(32.2)(4.14) = 258.62\ \frac{slug}{\cancel{ft^3}} \cdot \frac{\cancel{ft}}{s^2} \cdot ft$

$\rightarrow \boxed{\Delta p = 258.62\ \frac{slug}{ft \cdot s^2}}$ +15

3.

$\eta = 0.7 = \frac{\text{power in}}{\text{power out}} \rightarrow$ power in = 0.7 power out

power $= \rho g h_{pump} Q$

$Q = \frac{\pi}{4} D^2 V = \frac{\pi}{4}(.5^2)(6) = 1.18\ \frac{ft^3}{s}$

$\rightarrow$ power $= \frac{(1.94)(32.2)(4.14)(1.18)}{0.7} = 435.96\ \frac{slug}{\cancel{ft^3}} \cdot \frac{\cancel{ft}}{s^2} \cdot ft \cdot \frac{\cancel{ft^3}}{s}$

$\rightarrow \boxed{\text{power} = 435.96\ \frac{slug\ ft^2}{s^3}}$ +15

$+20/20$

QUESTION 2

Oil ($\rho = 950$ kg/m^3 and $\mu = 0.019$ kg/m.s) flows through a pipe of diameter D = 0.3 m and length L = 100 m. The pipe roughness length ratio ϵ/D = 0.0002.

The head loss due to friction is measured at 8 m.

Find the average velocity and the flow rate.

You may use either the *Direct Solution* or the *Iterative Solution*.

$\epsilon/D = 0.0002$
$D = 0.3$
$L = 100\,m$
$\rho = 950\ kg/m^3$
$\mu = 0.019\ kg/m{\cdot}s$
$h_f = 8\,m$

$$B = \frac{h_f D (2g)}{L} = \frac{(8)(0.3)(2\cdot 9.81)}{(100)} = 0.471\ \frac{m^2}{s^2}$$

$$h_f = \frac{8fL}{g\pi^2 D^5} Q^2 \rightarrow \frac{(8)(9.8)(\pi^2)(0.3^5)}{8(100)} = fQ^2 \rightarrow 0.00235 = fQ^2$$

$$(Q = \tfrac{\pi}{4} D^2 V) \rightarrow 0.00235 = f\left(\tfrac{\pi^2}{16}\right)(0.3^4)V^2$$

$$\rightarrow 0.47 = f V^2$$

$$\rightarrow f = \frac{0.47}{V^2} \quad (1)$$

(Iterative) $\rightarrow$ assume $f = 0.03$

$\rightarrow$ w/ $f = 0.03$, $\epsilon/D = 0.0002$ $\rightarrow h_f = f \frac{L}{D}\frac{V^2}{2g}$

$$\rightarrow V = \sqrt{\frac{h_f D (2g)}{fL}} = \sqrt{\frac{(8)(0.3)(19.6)}{(0.03)(100)}} = 3.96\ m/s \rightarrow Re = \frac{\rho V D}{\mu} = 5.9\cdot 10^4$$

$\rightarrow$ from Moody Chart with $Re = 6\cdot 10^4$, $\epsilon/D = 0.0002$ $\rightarrow f = 0.021$

$$\rightarrow h_f = f\frac{L}{D}\frac{V^2}{2g} \rightarrow V = \sqrt{\frac{h_f D (2g)}{fL}} = 4.73\ m/s \rightarrow Re = \frac{\rho V D}{\mu} = 7\cdot 10^4$$

$\rightarrow$ from Moody Chart with $Re = 7\cdot 10^4$, $\epsilon/D = 0.0002$ $\rightarrow f = 0.0205$

$$\rightarrow V = \sqrt{\frac{h_f D (2g)}{fL}} = 4.79\ m/s \rightarrow \boxed{V = 4.79\ m/s}$$

more iterations needed for a more precise solution

$$Q = \frac{\pi}{4} D^2 V = \frac{\pi}{4}(0.3^2)(4.79) = 0.339\ m^3/s$$

$$\rightarrow \boxed{Q = 0.339\ m^3/s}$$

+ 20/20

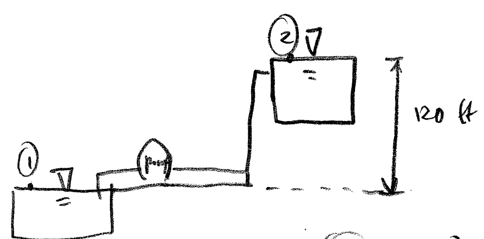
QUESTION 3

A pump is required to transport water between two reservoirs. The distance between the free surface levels in the two reservoirs is 120 ft.

The reservoirs are connected with a cast iron pipe of length L = 2000 ft, diameter D = 6 inch and roughness height ratio $\epsilon/D = 0.0017$.

A number of minor losses are present. The sum of all the minor loss coefficients is K = 2.5.

Calculate the power of the pump needed to pump water at a rate of 3 ft^3/s (assume 75% efficiency).



$L = 2000$ ft

$D = 6$ in $= 0.5$ ft

$\epsilon/D = 0.0017$

$K = 2.5$

$\eta = 0.75$

$Q = 3 \frac{ft^3}{s}$

$V = \frac{Q}{A} = \frac{3}{\left(\frac{\pi}{4}(0.5)^2\right)} = 15.3 \frac{ft}{s}$

B's (1)→(2): $\cancel{\frac{p_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + 0 + h_{pump} = \cancel{\frac{p_2}{\rho g}} + \cancel{\frac{V_2^2}{2g}} + 120 + h_f + h_{ml}$

(atm) (atm)

$h_{pump} = 120 + h_f$

$h_{pump} = 120 + \left(f\frac{L}{D} + K\right)\frac{V^2}{2g}$

$Re = \frac{\rho V D}{\mu} = \frac{(1.94)(15.3)(0.5)}{2.09 \cdot 10^{-5}} = 7.1 \cdot 10^5 \longrightarrow$ use Moody Chart $\rightarrow f = 0.0225$ +15

$h_{pump} = 120 + \left(0.0225\left(\frac{2000}{0.5}\right) + 2.5\right)\frac{(15.3)^2}{(64.4)} = 456.23$ ft

$\text{power} = \frac{\rho g h_{pump} Q}{0.75} = \frac{(1.94)(32.2)(456.23 \text{ ft})(3)}{0.75} = \boxed{114{,}000 \frac{\text{slug ft}^2}{s^3}}$

QUESTION 4

Water flows along a rectangular channel at a depth of 1.3 m. The channel's width is 5.5 m, its slope is 0.001, and has a Manning's n of 0.024.

A rectangular bump is placed on the bed of this channel some distance after uniform flow is established.

Calculate:

1. the height of this bump required to produce **critical** flow on the crest,
2. the maximum depression in the water-surface level above the bump.

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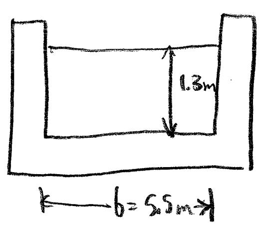


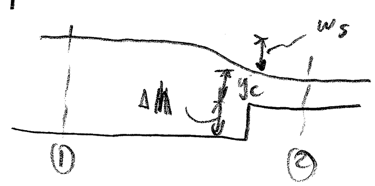
$S_0 = 0.001$

$n = 0.024$

$C_m = 1$

$A = (5.5)(1.3) = 7.15 m^2$

$R_h = \frac{A}{P} = \frac{7.15}{(1.3 \cdot 2) + 5.5} = 0.883$



1.

$Q = \frac{C_m}{n} A R^{2/3} S_0^{1/2} = \frac{1}{0.024}(7.15)(0.883)^{2/3}(0.001)^{1/2} = 8.67 \frac{m^3}{s}$

$A_c = \left(\frac{b_c Q^2}{g}\right)^{1/3} = \left(\frac{(5.5)(8.67^2)}{(9.8)}\right)^{1/3} = 3.48 m^2$

$\frac{Q}{b} = q \rightarrow \frac{8.67}{5.5} = 1.576 \rightarrow q = 1.576 \frac{m^2}{s}$ +1

$Q = A_1 V_1 = A_c V_c \rightarrow V_c = \frac{Q}{A_c} = 2.49 \frac{m}{s}$ +3 $\rightarrow V_1 = 1.21 \frac{m}{s}$ +5

$\frac{A_c}{b_c} = y_c \rightarrow y_c = \frac{3.48 m^2}{5.5 m} = 0.63 m$ +3

$E_1 = E_2 + \Delta h \rightarrow y_1 + \frac{V_1^2}{2g} = y_c + \frac{V_c^2}{2g} + \Delta h$ +5 $\rightarrow 1.375 m = 0.946 m + \Delta h$

$\rightarrow \boxed{\Delta h = 0.429 m}$

2. $y_1 = \Delta h + y_c + w_s \rightarrow w_s = y_1 - \Delta h - y_c = 1.3 - 0.429 - 0.63 = .241$

$\boxed{\text{depression in water surface} \leq 0.241 m}$ +3

QUESTION 5

A hydraulic jump is induced in an 80 ft wide channel.

The water depths on either side of the jump are 1 ft and 10 ft.

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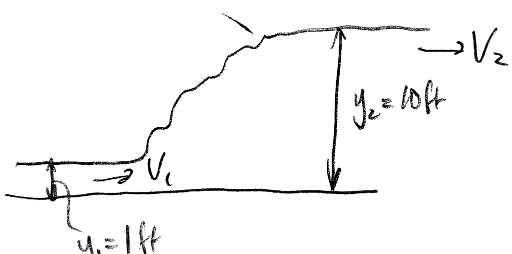
Calculate:

1. The velocity of the faster moving flow,
2. The flow rate,
3. The Froude number of the sub–critical flow,
4. The flow energy dissipated in the hydraulic jump (expressed as percentage of the energy prior to the jump).



$b = 80 ft$

1. $$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{1+8Fr_1^2} - 1\right) \to \sqrt{\frac{\left(\frac{2y_2}{y_1}+1\right)^2 - 1}{8}} = Fr_1$$

$$\to Fr_1 = \sqrt{\frac{(2(10)+1)^2 - 1}{8}} = 7.42$$

+5

$$Fr_1 = \frac{V_1}{\sqrt{y_1 g}} \to V_1 = Fr\sqrt{y_1 g} = (7.42)\sqrt{1 \cdot (32.2)} = \boxed{42.1 \frac{ft}{s}}$$

2. $$Q = VA = V_1 A_1 = (42.1 \tfrac{ft}{s})(1 \cdot 80) = \boxed{3368 \frac{ft^3}{s}}$$ +5

3. $$Fr_2 = \sqrt{\frac{\left(2\left(\frac{y_1}{y_2}\right)+1\right)^2 - 1}{8}} = \boxed{0.235}$$ +5

$E_2 \approx 10.3$

4. $$E_1 = E_2 + \Delta E \to y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + \Delta E \to \Delta E = \overset{E_1}{28.5} - \overset{E_2}{0.275} = 28.22$$

+2 $$\% \text{ dissipated} = \frac{\Delta E}{E_1} = \frac{28.22}{28.5} \cdot 100 = \boxed{99\%}$$