ECI 141: ENGINEERING HYDRAULICS

MID-TERM EXAMINATION

SPRING 2011

NAME: SOLUTION SHEETS

PLEASE ANSWER ONLY 5 OUT OF THE 6 QUESTIONS:
For water at 20°C, please take $\rho=10^3$ kg/m^3 (=1.94 slug/ft^3) and $\mu=10^{-3}$ kg/m s (=2.09 x 10^{-5} slug/ft s). All questions worth the same number of points (20/100).

QUESTION 1 (pipe flow; energy losses)

A horizontal pipe conveys water (20° C) at a rate of 0.5 ft^3/s. a) Please compute the energy loss for the following values: D=5 inches, L=300 ft, roughness height=$5x10^{-4}$ ft. b) Please determine the regime of the flow (laminar, turbulent smooth, turbulent fully-rough or turbulent transitional). c) Please repeat the computations for a flow rate of 5 x 10^{-2} ft^3/s.

a) $Q = 0.5\, ft^3/s \Rightarrow V = \frac{Q}{A} = \frac{0.5\, ft^3/s \times 4}{\pi (5/12)^2 ft^2} = \underline{3.67\, ft/s}$

$h_f = f \frac{L}{D} \frac{V^2}{2g}$ Darcy-Weisbach

$h_f = ?$

$Re = \frac{VD}{\nu} = \frac{3.67\, ft/s\, (5/12)\, ft}{\frac{2.09 \times 10^{-5}\, slug/ft\,s}{1.94\, slug/ft^3}} = 1.42 \times 10^5$

$\nu \uparrow \mu/\rho$

$\frac{k_s}{D} = \frac{5 \times 10^{-4}\, \cancel{ft}}{5/12\, \cancel{ft}} = 1.2 \times 10^{-3}$

$\Rightarrow$

from Moody chart: $f = 0.022$

$\Rightarrow \boxed{h_f = 0.022 \times \frac{300\, \cancel{ft}}{5/12\, \cancel{ft}} \frac{(3.67)^2\, \cancel{ft^2/s^2}}{2 \times 32.2\, \cancel{ft/s^2}} = 3.32\, ft}$

b) Regime: turbulent/transitional

c) $V = \frac{Q}{A} = \frac{5 \times 10^{-2}\, ft^3/s \times 4}{\pi\, (5/12)^2} = 0.37\, ft/s$ (as expected)

$Re = 1.42 \times 10^4$

$\frac{k_s}{D} = 1.2 \times 10^{-3}$

$\Rightarrow f = 0.03$

$\Rightarrow \boxed{h_f = 0.03 \times \frac{300\, \cancel{ft}}{5/12\, \cancel{ft}} \frac{(0.37)^2}{2 \times 32.2}\, ft = 0.046\, ft}$

Also transitional

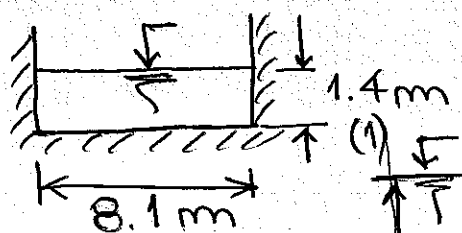
QUESTION 2 (open-channel flow; critical flow)

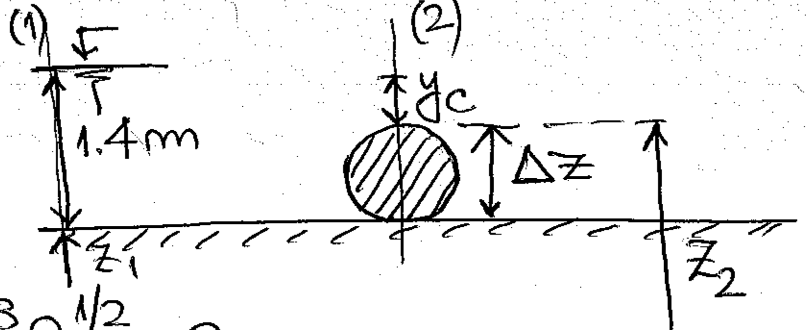
Water flows along a rectangular channel at a depth of 1.4 m. The channel's width is 8.1 m, its slope is 0.002 and has a Manning's coefficient *n* of 0.018.

A tree falls into the channel and lies on the bed of this channel some distance after uniform flow is established. Please treat the tree as a cylinder located in the transverse direction of the channel. Calculate the diameter of the tree knowing that it produced the **critical flow** on top of the tree.

$S_0 = 0.002$

$m = 0.018$





Using Manning's

$$V = \frac{K_m R^{2/3} S_0^{1/2}}{m} = ?$$

$$R = \frac{A}{P} = \frac{1.4\,m \times 8.1\,\cancel{m}}{(8.1 + 2\times 1.4)\,\cancel{m}} = 1.04\,m$$

$$V = \frac{(1.04)^{2/3}\; 0.002^{1/2}}{0.018}\,m/s = 2.55\,m/s$$

$$\Rightarrow Q = VA = 2.55\,m/s \times 1.4\,m \times 8.1\,m = 28.92\,m^3/s$$

Using Bernoulli Eqn. (no losses)

$$z_1 + y_1 + \frac{V_1^2}{2g} = y_2 + \frac{V_2^2}{2g} + z_2$$

$$E_1 = E_2 + z_2 - z_1 = E_2 + \Delta z$$

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{2.55\,m/s}{\sqrt{9.81\,\frac{m}{s^2} \times 1.4\,m}} = 0.69 < 1$$

subcritical

$$E_2 = ? = y_c + \frac{v_c^2}{2g}$$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{(28.92\,m^3/s)^2}{(8.1)^2 m^2 \times 9.81\,m/s^2}} = 1.091\,m$$

$$v_c = \frac{Q}{A_c} = \frac{28.92\ m^3/s}{1.091\,m \times 8.1\,m} = 3.27\,m/s$$

$$\Rightarrow E_2 = E_c = y_c + \frac{v_c^2}{2g} = 1.091\,m + \frac{(3.272)^2}{2 \times 9.81}\,m = 1.637\,m$$

$$E_1 = y_1 + \frac{v_1^2}{2g} = 1.4\,m + \frac{(2.55)^2}{2 \times 9.81}\,m = 1.731\,m$$

$$\Rightarrow \boxed{\Delta z = E_1 - E_2 = 0.095\,m}$$

$\Rightarrow$ it is a small tree indeed

QUESTION 3 (pipe flow; energy losses)

Oil (ρ = 900 Kg/m^3 and μ=0.02 kg/m s) flows through a pipe of diameter D=0.25 m and length L=200 m. The pipe roughness length ratio ε/D=0.0005. a) If the head loss is 16.15 m, please compute the discharge flowing through the pipe. b) Please compute the pipe roughness length ratio for Q=0.15 m^3/s.

$$h_f = 16.15 \text{ m}$$

$$\nu = \frac{\mu}{\rho} = \frac{0.02 \frac{\cancel{kg}}{\cancel{m}\, s}}{900 \frac{\cancel{kg}}{m^{\cancel{3}2}}} = 2.22 \times 10^{-5} \frac{m^2}{s}$$

$$h_f = f \frac{L}{D} \frac{V^2}{2g}$$

a) Using Direct method by Rouse:

$$B = \frac{h_f g D^3}{L \nu^2} = \frac{16.15 \cancel{m} \times 9.81 \cancel{m}/\cancel{s^2} \times (0.25)^3 \cancel{m^3}}{200 \cancel{m} \, (2.22 \times 10^{-5})^2 \cancel{m^4}/\cancel{s^2}} =$$

$$2.506 \times 10^7$$

$$\Rightarrow Re = -(8B)^{1/2} \log_{10}\left[\frac{\varepsilon/D}{3.7} + \frac{1.775}{\sqrt{B}}\right]$$

$$= -(8 \times 2.506 \times 10^7)^{1/2} \log_{10}\left[\frac{5 \times 10^{-4}}{3.7} + \frac{1.775}{\sqrt{2.506 \times 10^7}}\right]$$

$$= 4.9 \times 10^4$$

$$Re = \frac{VD}{\nu} \Rightarrow V = \frac{Re\, \nu}{D} = \frac{4.9 \times 10^4 \times 2.22 \times 10^{-5} \, m^2/s}{0.25 \text{ m}}$$

$$= 4.37 \text{ m/s}$$

$$\Rightarrow \boxed{Q = VA = 4.37 \frac{m}{s} \frac{\pi (0.25)^2}{4} m^2 = 0.214 \frac{m^3}{s}}$$

b) $Q = 0.15 \text{ m}^3/\text{s}$

$$\Rightarrow V = \frac{Q}{A} = \frac{0.15\ m^3/s \times 4}{\pi (0.25)^2\ m^2} = 3.056\ m/s$$

$$\Rightarrow f = \frac{h_f\ D\, 2g}{L\, V^2} = \frac{16.15\ m \times 0.25\ m \times 2 \times 9.81\ m/s^2}{200\ m \times (3.056)^2\ m^2/s^2} =$$

0.0424

$$Re = \frac{VD}{\nu} = \frac{3.056\ m/s \times 0.25\ m}{2.22 \times 10^{-5}\ m^2/s} = 3.4 \times 10^4$$

$\Rightarrow$ from Moody chart $\boxed{\frac{k_s}{D} = \frac{\varepsilon}{D} = 0.013}$

QUESTION 4 (pipe flow; open-channel flow; hydraulic jump)

Now please assume that the pipe of Question 1 discharges to an open channel of width b=0.25 ft, keeping a water depth equal to the diameter. What is the sequent depth y_2 in order to form a free hydraulic jump downstream of the pipe discharge?

$$\left.\begin{array}{l} Q = 0.5\ ft^3/s \\ b = 0.25\ ft \end{array}\right\} q = \frac{Q}{b} = \frac{0.5\ ft^3/s}{0.25\ ft} = 2\ ft^2/s$$

$$V_1 = \frac{q}{y_1} = \frac{2\ ft^2/s}{5/12\ \cancel{ft}} = 4.8\ ft/s$$

$$Fr_1 = \frac{V_1}{\sqrt{g y_1}} = \frac{4.8\ ft/s}{\sqrt{32.2 \times 5/12\ \frac{ft}{s^2}\ ft}} = 1.31 > 1$$

super-critical

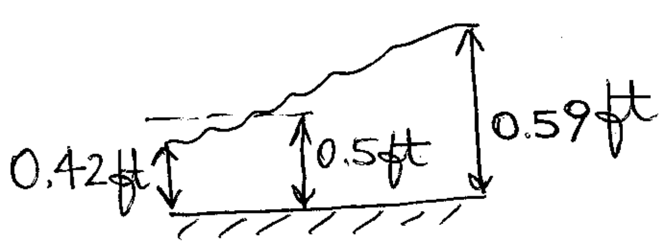
$$\frac{y_2}{y_1} = \frac{1}{2}\left(\sqrt{8 Fr_1^2 + 1} - 1\right) =$$

$$\frac{1}{2}\left(\sqrt{8 \times 1.31^2 + 1} - 1\right) \Rightarrow 1.42 = y_2/y_1$$

$$\Rightarrow \boxed{y_2 = 0.59\ ft}$$

$$Fr_2 = \frac{q/y_2}{\sqrt{g y_2}} = \frac{2\ ft^2/s / 0.591\ ft}{\sqrt{32.2\ ft/s^2 \times 0.591\ ft}} = 0.78 < 1$$

$$y_c = \sqrt[3]{\frac{q^2}{g}} = \sqrt[3]{\frac{4\ ft^4/s^2}{32.2\ ft/s^2}} = 0.5\ ft$$



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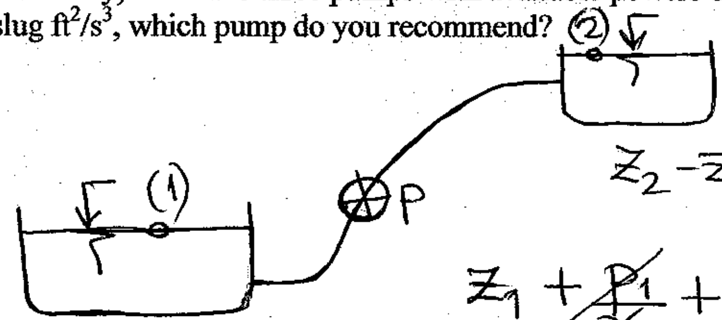
QUESTION 5 (pipe flow; power; energy losses)

A pump is required to transport water between two reservoirs. The distance between the free surface levels in the two reservoirs is 150 ft.

The reservoirs are connected with a cast iron pipe of length L=3000 ft, diameter D = 10 inches and roughness ratio ε/D=0.0012.

A number of minor losses are present. The sum of all the minor loss coefficients is K=10.5.

Calculate the power of the pump needed to pump water at a rate of 5 ft^3/s (assume 65% efficiency). If I have three pumps with available powers of 100,000, 150,000 and 200,000 slug ft^2/s^3, which pump do you recommend?



$z_2 - z_2 = 150\ ft$

$z_1 + \cancel{\frac{p_1}{\gamma}} + \cancel{\frac{V_1^2}{2g}} + h_{pump} =$

Patm Large reservoir

$z_2 + \cancel{\frac{p_2}{\gamma}} + \cancel{\frac{V_2^2}{2g}} + h_f + \sum h_m$

$$\Rightarrow h_{pump} = z_2 - z_1 + h_f + \sum h_m$$

$$h_{pump} = 150\ ft + f \frac{L}{D}\frac{V^2}{2g} + \sum K \frac{V^2}{2g}$$

$$Q = 5\ ft^3/s \Rightarrow V = \frac{Q}{A} = \frac{5\ ft^3/s \times 4}{\pi (10/12)^2 ft^2} = 9.167\ ft/s$$

$$\Rightarrow \quad Re = \frac{VD}{\nu} = 7.1 \times 10^5$$

$$\frac{V^2}{2g} = \frac{(9.167)^2\ ft^2/s^2}{2 \times 32.2\ ft/s^2} = 1.305\ ft \ ; \ \frac{k_s}{D} = 0.0012 \qquad f = 0.021$$

$$h_{pump} = 150\ ft + 1.305\ ft \left[0.021 \times \frac{3000\ ft}{(10/12)\ ft} + 10.5\right]$$

$$= 262.36\ ft$$

$$Power = \rho g h_p Q/\eta = \frac{1.94\ slug/ft^3 \times 32.2\ ft/s^2 \times}{0.65}$$

$\times\ 262.36\ ft \times 5\ ft^3/s = 126{,}070\ slug\ \frac{ft}{s^2}\ \frac{ft}{s}$

Then, I select the pump with $\underline{\underline{150{,}000\ slug\ \frac{ft^2}{s^3}}}$

QUESTION 6 (pipe flow; energy losses)

A cast-iron horizontal pipe is used to carry water (ε=0.0005 ft). Determine the velocity of water transported in the pipe when the head loss between points separated by L=200 ft is 1.54 ft. The pipe has a diameter D=6 inches.

Calculate also the input power to the pump required to convey the water (assume pump efficiency of 85%).

$$\nu = \frac{\mu}{\rho} = \frac{2.09\times 10^{-5}\ \cancel{slug}/\cancel{ft}/s}{1.94\ \cancel{slug}/\cancel{ft^3}} = 1.077\times 10^{-5}\ ft^2/s$$

$$B = \frac{h_f\, g\, D^3}{L\,\nu^2} = \frac{1.54\,\cancel{ft}\times 32.2\,\cancel{ft/s^2}\,(6/12)^3\,ft^3}{200\,\cancel{ft}\times(1.077\times10^{-5})^2\,\cancel{ft^4/s^2}} =$$

$$267\,193\,078.1 = 2.67\times10^8$$

$$Re = -(8B)^{1/2}\log_{10}\left[\frac{ks/D}{3.7} + \frac{1.775}{\sqrt{B}}\right] =$$

$$-(8\times 2.67\times10^8)^{1/2}\log_{10}\left[\frac{5\times10^{-4}/0.5}{3.7} + \frac{1.775}{\sqrt{2.67\times10^8}}\right]$$

$$= 1.58\times10^5 = V D/\nu$$

$$\Rightarrow \boxed{V} = \frac{Re\ \nu}{D} = \frac{1.58\times10^5\times1.077\times10^{-5}\ \cancel{ft}^2/s}{0.5\ \cancel{ft}}$$

$$\boxed{3.41\ ft/s} \Rightarrow Q = VA = 0.67\ ft^3/s$$

$$\boxed{Power} = \frac{\rho g Q h_f}{\eta} = \frac{1.94\frac{slug}{ft^3}\times 32.2\frac{ft}{s^2}\times 0.67\ ft^3/s}{0.85}$$

$$\times 1.54\ ft = \boxed{75.82\ slug\ ft^2/s^3}$$