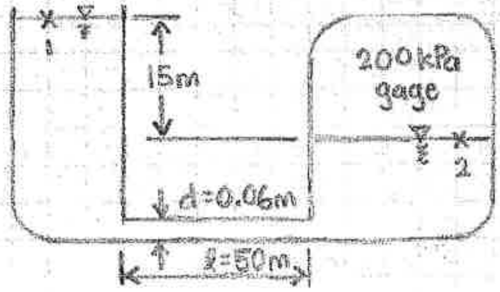
Problem 7 6.78 In the figure below the connecting pipe is commercial steel 6cm in diameter. Estimate the flow rate, in m^3/hr, if the fluid is water at 20°C. Which way is the flow?



Water at 20°C (Table A.1. White 7th ed.)

$\rho = 998\ \frac{kg}{m^3}$; $\nu = 1.005\times10^{-6}\ \frac{m^2}{s}$

commercial steel (Table 6.1. White 7th ed.)

$\epsilon = 4.6\times10^{-5} m$

$\frac{\epsilon}{d} = \frac{4.6\times10^{-5}m}{0.06m} = 7.667\times10^{-4}$

Bernoulli's from point 1 to 2:

$$\cancel{\frac{P_1}{\rho g}} + \cancel{\frac{V_1^2}{2g}} + z_1 = \frac{P_2}{\rho g} + \cancel{\frac{V_2^2}{2g}} + z_2 + h_f$$

atmospheric pressure; ≈ 0 b/c of large reservoir; ≈ 0 b/c of large tank

$V_1 = V_2 \approx 0$ (b/c of large water body)

$P_1 = P_{atm} = 0$

$$h_f = z_1 - z_2 - \frac{P_2}{\rho g} = 15m - 0m - \frac{200\times10^3\ kg/m\cdot s^2}{998\ kg/m^3\,(9.81\ m/s^2)} = -5.428\ m \quad \boxed{\text{flow to left} \leftarrow}$$

$$h_f = f\,\frac{l}{d}\cdot\frac{V^2}{2g} \Rightarrow V = \sqrt{\frac{2gd\cdot h_f}{f\,l}}$$

Assume turbulent flow so for $\frac{\epsilon}{d} = 7.667\times10^{-4}$, $f \approx 0.018$

Iterate to find f:

1. $V = \sqrt{\frac{2(9.81\ m/s^2)(0.06m)(5.428m)}{0.018(50m)}} = 2.665\ \frac{m}{s}$

$Re = \frac{Vd}{\nu} = \frac{2.665\ m/s\,(0.06m)}{1.005\times10^{-6}\ m^2/s} = 159100$

from the Moody-Chart: $f_1 = 0.0205$

2. $V = \sqrt{\frac{2(9.81\ m/s^2)(0.06m)(5.428m)}{0.0205(50m)}} = 2.497\ \frac{m}{s}$

$Re = \frac{2.497\ m/s\,(0.06m)}{1.005\times10^{-6}\ m^2/s} = 149070$

from the Moody-Chart: $f_2 = 0.0205 = f_1$ converged!

$$Q = VA = 2.497\ \frac{m}{s}\left(\pi\left(\frac{0.06m}{2}\right)^2\right)\left(\frac{3600s}{hr}\right) \Rightarrow \boxed{Q = 25.42\ \frac{m^3}{hr}} \leftarrow$$

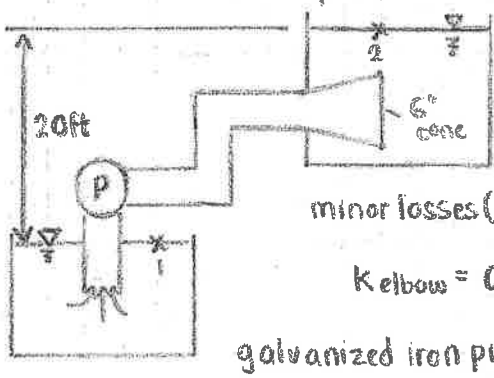
gage pressure:

The pressure relative to the local atmospheric pressure.

gage pressure $p > p_{atm}$

$p(gage) = p - p_a$

Problem 9 6.102. A 70% efficient pump delivers water at 20°C from one reservoir to another 20 ft higher. The piping system consists of 60 ft of galvanized iron 2 in pipe, a reentrant entrance, two screwed 90° long-radius elbows, a screwed-open gate valve, and a sharp exit. What is the input power required in horsepower with and without a 6° well-designed conical expansion added to the exit? The flow rate is 0.4 ft³/s.



Water at 20°C (Table A.1. White 7th ed.):

$\rho = 1.937\ \text{slugs/ft}^3$, $\nu = 1.082 \times 10^{-5}\ \text{ft}^2/\text{s}$

minor losses (Table 6.5. White 7th ed.): $K_{entry} = 1.0$;

$K_{elbow} = 0.41$; $K_{valve} = 0.16$; $K_{exit} = 1.0$; $K_{cone} = 0.3$

galvanized iron pipe (Table 6.1 White 7th ed.): $\epsilon = 0.0005\ \text{ft}$

Bernoulli's from point 1 to 2:

$$\underbrace{\cancel{\frac{P_1}{\rho g}}}_{\text{atmospheric pressure}} + \underbrace{\cancel{\frac{V_1^2}{2g}}}_{\approx 0 \text{ b/c of large reservoir}} + z_1 + h_p = \underbrace{\cancel{\frac{P_2}{\rho g}}}_{\text{atmospheric pressure}} + \underbrace{\cancel{\frac{V_2^2}{2g}}}_{\approx 0 \text{ b/c of large reservoir}} + z_2 + h_f + h_{ml}$$

$V_1 = V_2 \approx 0$ (large reservoir)

$P_1 = P_2 = P_{atm}$

$$\Rightarrow \quad h_p = z_2 - z_1 + h_f + h_{ml}$$

$$A = \pi r^2 = \pi\left(\frac{2\text{in}}{2} \cdot \frac{1\text{ft}}{12\text{in}}\right)^2 = 0.0218\ \text{ft}^2 \quad ; \quad V = \frac{Q}{A} = \frac{0.4\ \text{ft}^3/\text{s}}{0.0218\ \text{ft}^2} = 18.335\ \frac{\text{ft}}{\text{s}}$$

$$Re = \frac{Vd}{\nu} = \frac{18.335\ \text{ft/s} \cdot 2\text{in}\left(1\text{ft}/12\text{in}\right)}{1.082 \times 10^{-5}\ \text{ft}^2/\text{s}} = 282420 \ ; \quad \frac{\epsilon}{d} = \frac{0.0005\ \text{ft}}{2\text{in}\left(1\text{ft}/12\text{in}\right)} = 0.003$$

From Moody-Chart: $f = 0.0265$

$$h_f = f\frac{l}{d}\frac{V^2}{2g} = 0.0265 \cdot \frac{60\text{ft}}{2\text{in}\left(1\text{ft}/12\text{in}\right)} \cdot \frac{(18.335\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = 49.8\ \text{ft}$$

1. without a 6° cone expansion:

$$h_{ml} = \Sigma k \frac{V^2}{2g} = \left(K_{entry} + 2K_{elbow} + K_{valve} + K_{exit}\right)\frac{V^2}{2g}$$

$$= (1.0 + 2(0.41) + 0.16 + 1.0)\ \frac{(18.335\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = 15.56\ \text{ft}$$

$$h_p = z_2 - z_1 + h_f + h_{ml}$$

$$= 20\text{ft} - 0\text{ft} + 49.8\text{ft} + 15.56\text{ft} = 85.36\text{ ft}$$

$$\text{Power } P = \frac{\rho g Q h_p}{\eta} = \frac{1.937\,\text{slugs/ft}^3(32.2\,\text{ft/s}^2)(0.4\,\text{ft}^3/\text{s})(85.36\,\text{ft})}{0.70} = 3042.3\,\frac{\text{lbf}\cdot\text{ft}}{\text{s}}$$

$$\text{Power} = 3042.3\,\frac{\text{lbf}\cdot\text{ft}}{\text{s}}\left(\frac{1\text{ hp}}{550\,\text{lbf}\cdot\text{ft/s}}\right) \Rightarrow \boxed{P = 5.53\text{ hp}}$$

2. with a 6° cone expansion:

$$h_{ml} = \sum k\frac{V^2}{2g} = (K_{entry} + 2K_{elbows} + K_{valve} + K_{cone})\frac{V^2}{2g}$$

$$= (1.0 + 2(0.41) + 0.16 + 0.3)\frac{(18.335\,\text{ft/s})^2}{2(32.2\,\text{ft/s}^2)} = 11.90\text{ ft}$$

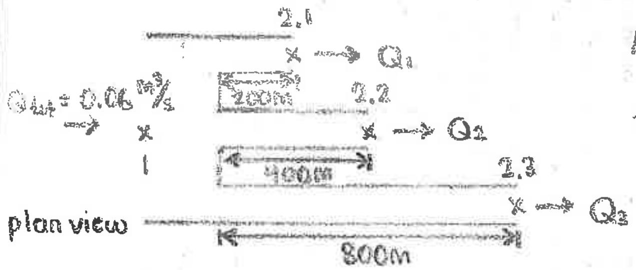
$$h_p = z_2 - z_1 + h_f + h_{ml}$$

$$= 20\text{ft} - 0\text{ft} + 49.8\text{ft} + 11.90\text{ft} = 81.7\text{ ft}$$

$$\text{Power } P = \frac{\rho g Q h_p}{\eta} = \frac{1.937\,\text{slugs/ft}^3(32.2\,\text{ft/s}^2)(0.4\,\text{ft}^3/\text{s})(81.7\,\text{ft})}{0.70} = 2911.85\,\frac{\text{lbf}\cdot\text{ft}}{\text{s}}$$

$$\text{Power} = 2911.85\,\frac{\text{lbf}\cdot\text{ft}}{\text{s}}\left(\frac{1\text{hp}}{550\,\text{lbf}\cdot\text{ft/s}}\right) \Rightarrow \boxed{P = 5.29\text{ hp}}$$

Problem 10 Three smooth pipes with diameter $D = 10\,cm$ and lengths $L_1 = 200\,m$, $L_2 = 400\,m$, $L_3 = 800\,m$ are laid in parallel and are supplied by water at a rate of $0.06\,\frac{m^3}{s}$. Please determine: a) The flow rate in each pipe; b) the pressure drop across the system. Please start the iterations with a value of the friction factor equal to 0.02 for the three pipes.



Assume water at 20°C (Table A.1. White 7th ed.)

$\rho = 998\,\frac{kg}{m^3}$; $\nu = 1.005\times10^{-6}\,\frac{m^2}{s}$

$d = 10\,cm\left(\frac{10^{-2}\,m}{1\,cm}\right) = 0.1\,m$

$A = \pi r^2 = \pi\left(\frac{0.1\,m}{2}\right)^2 = 0.00785\,m^2$

$Q = V\cdot A$

$Q_{in} = Q_{out}$; continuity

$Q_{tot} = Q_1 + Q_2 + Q_3$

Bernoulli's from point 1 to 2:

$$\frac{P_1}{\rho g} + \underbrace{\cancel{\frac{V_1^2}{2g}}}_{\substack{Q_{in}=Q_{out}\\ V_1=\Sigma V_i}} + \underbrace{\cancel{z_1}}_{\substack{\text{pipes are}\\ \text{@ equal height}}} = \frac{P_2}{\rho g} + \underbrace{\cancel{\frac{V_2^2}{2g}}}_{\substack{Q_{in}=Q_{out}\\ V_1=\Sigma V_i}} + \underbrace{\cancel{z_2}}_{\substack{\text{pipes are}\\ \text{@ equal height}}} + h_f$$

$$\Rightarrow \Delta p = \rho g h_f$$

$$c_i = \frac{\pi^2 g d_i^5}{8 l_i} : \quad c_1 = \frac{\pi^2(9.81\,m/s^2)(0.1\,m)^5}{8(200\,m)} = 6.05\times10^{-7}\,\frac{m^5}{s^2}$$

$$c_2 = \frac{\pi^2(9.81\,m/s^2)(0.1\,m)^5}{8(400\,m)} = 3.025\times10^{-7}\,\frac{m^5}{s^2}$$

$$c_3 = \frac{\pi^2(9.81\,m/s^2)(0.1\,m)^5}{8(800\,m)} = 1.513\times10^{-7}\,\frac{m^5}{s^2}$$

Iterate to find h_f, V, Re, when it converges find Q

$$h_f = \frac{Q_{tot}^2}{\left(\Sigma\sqrt{c_i/f_i}\right)^2} \; ; \quad V_i = \sqrt{\frac{h_f\cdot d\cdot 2g}{l_i f_i}} \; ; \quad Re_i = \frac{d V_i}{\nu}$$

1. $f_1 = f_2 = f_3 = 0.02$

$$h_f = \frac{(0.06\,m^3/s)^2}{\left(\sqrt{6.05\times10^{-7}\,m^5/s^2/0.02} + \sqrt{3.025\times10^{-7}\,m^5/s^2/0.02} + \sqrt{1.513\times10^{-7}\,m^5/s^2/0.02}\right)^2} = 24.43\,m$$

$$V_1 = \sqrt{\frac{24.43m(0.1m)2(9.81 m/s^2)}{200m(0.02)}} = 3.46 \frac{m}{s}$$

$$V_3 = \sqrt{\frac{24.43m(0.1m)2(9.81 m/s^2)}{800m(0.02)}} = 1.73 \frac{m}{s}$$

$$V_2 = \sqrt{\frac{24.43m(0.1m)2(9.81 m/s^2)}{400m(0.02)}} = 2.45 \frac{m}{s}$$

$$Re_1 = \frac{0.1m(3.46 m/s)}{1.005\times10^{-6} m^2/s} = 344280$$

$$Re_3 = \frac{0.1m(1.73 m/s)}{1.005\times10^{-6} m^2/s} = 172140$$

$$Re_2 = \frac{0.1m(2.45 m/s)}{1.005\times10^{-6} m^2/s} = 243800$$

From the Moody-Chart : $f_1 = 0.0138$; $f_2 = 0.0146$; $f_3 = 0.0156$

2.
$$h_f = \frac{(0.06 m^3/s)^2}{(\sqrt{6.05\times10^{-7} m^5/s^2 / 0.0138} + \sqrt{3.025\times10^{-7} m^5/s^2 / 0.0146} + \sqrt{1.513\times10^{-7} m^5/s^2 / 0.0156})^2} = 17.64 m$$

$$V_1 = \sqrt{\frac{17.64m(0.1m)2(9.81 m/s^2)}{200m(0.0138)}} = 3.54 \frac{m}{s}$$

$$V_3 = \sqrt{\frac{17.64m(0.1m)2(9.81 m/s^2)}{800m(0.0156)}} = 1.67 \frac{m}{s}$$

$$V_2 = \sqrt{\frac{17.64m(0.1m)2(9.81 m/s^2)}{400m(0.0146)}} = 2.43 \frac{m}{s}$$

$$Re_1 = \frac{0.1m(3.54 m/s)}{1.005\times10^{-6} m^2/s} = 352238$$

$$Re_3 = \frac{0.1m(1.67 m/s)}{1.005\times10^{-6} m^2/s} = 166169$$

$$Re_2 = \frac{0.1m(2.43 m/s)}{1.005\times10^{-6} m^2/s} = 241791$$

From the Moody Chart : $f_1 = 0.0136$; $f_2 = 0.0146$, $f_3 = 0.0158$

3.
$$h_f = \frac{(0.06 m^3/s)^2}{(\sqrt{6.05\times10^{-7} m^5/s^2 / 0.0136} + \sqrt{3.025\times10^{-7} m^5/s^2 / 0.0146} + \sqrt{1.513\times10^{-7} m^5/s^2 / 0.0158})^2} = 17.57 m$$

$$V_1 = \sqrt{\frac{17.57m(0.1m)2(9.81 m/s^2)}{200m(0.0136)}} = 3.56 \frac{m}{s}$$

$$V_3 = \sqrt{\frac{17.57m(0.1m)2(9.81 m/s^2)}{800m(0.0158)}} = 1.65 \frac{m}{s}$$

$$V_2 = \sqrt{\frac{17.57m(0.1m)2(9.81 m/s^2)}{400m(0.0146)}} = 2.43 \frac{m}{s}$$

$$Re_1 = \frac{0.1m(3.56 m/s)}{1.005\times10^{-6} m^2/s} = 354228$$

$$Re_3 = \frac{0.1m(1.65 m/s)}{1.005\times10^{-6} m^2/s} = 164179$$

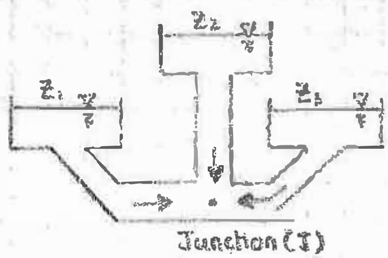
$$Re_2 = \frac{0.1m(2.43 m/s)}{1.005\times10^{-6} m^2/s} = 241791$$

From the Moody Chart : $f_1 = 0.0136$; $f_2 = 0.0146$; $f_3 = 0.0158$ converged!

$Q_1 = 3.56 \frac{m}{s}(0.00785 m^2) = 0.028 \frac{m^3}{s}$; $Q_2 = 2.43 \frac{m}{s}(0.00785 m^2) = 0.019 \frac{m^3}{s}$

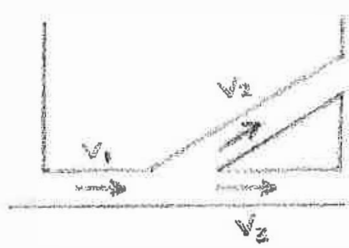
$Q_3 = 1.65 \frac{m}{s}(0.00785 m^2) = 0.013 \frac{m^3}{s}$; $\Delta p = 998 \frac{kg}{m^3}(9.81 \frac{m}{s^2})(17.57 m) = 172016 \frac{kg}{m \cdot s^2}$

Problem 11 Compare the treatments for the case of flow at a junction found in the book (White, 2011), and in the book by Munson et al. Please describe briefly the similarities and differences. Please discuss the hypothesis associated with each approach.



White, 2011

- The problem is solved at the junction by assuming $Q_1 + Q_2 + Q_3 = 0$, and $h_J = z_J + \frac{P_J}{\rho g}$
- Guess h_J and iterate until $Q_1 + Q_2 + Q_3 = 0$



Munson et al.

- They used Bernoulli's to solve V_1, V_2, and V_3
- Developed a system of 3 algebraic equations & 3 unknowns to solve for V_1, V_2, and V_3
- They then checked assumptions for flow direction

Problem 13 Using the conditions of Problem 7, please compute Manning's n of such a pipe.

Manning's equation: $Q = \frac{\alpha}{n} A R_h^{2/3} S_o^{1/2}$ where $\alpha = 1$ for S.I. units and $\alpha = 1.486$ for English units

$$R_h = \frac{A}{P} = \frac{\pi (d/2)^2}{\pi d} = \frac{d}{4}$$

$$S_o \Rightarrow S_f = \frac{h_f}{\ell}$$

From Problem 7:

$Q = 0.00706 \frac{m^3}{s}$; $d = 0.06m$; $A = \pi\left(\frac{0.06m}{2}\right)^2 = 0.00283 m^2$;

$h_f = 5.428m$; $\ell = 50m$

$$R_h = \frac{d}{4} = \frac{0.06m}{4} = 0.015m$$

$$S_o \Rightarrow S_f = \frac{h_f}{\ell} = \frac{5.428m}{50m} = 0.10856$$

$$n = \frac{\alpha}{Q} A R_h^{2/3} S_o^{1/2} = \frac{1}{0.00706 m^3/s} \cdot 0.00283m^2 (0.015m)^{2/3} (0.10856)^{1/2}$$

$$\Rightarrow \boxed{n = 0.008}$$