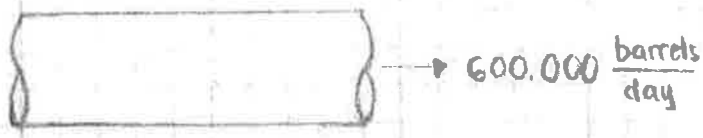
Problem 3 6.2. The present pumping rate of North Slope crude oil through the Alaska Pipeline is about 600,000 barrels per day (1 barrel = 42 U.S. gallons). What would be the maximum rate if the flow were constrained to be laminar? Assume that Alaskan oil is crude oil in Fig A.1. of the Appendix at 60°C.



crude oil at 60°C (Fig A.1. White 7th ed.):

$\mu = 4\times10^{-3}\ \frac{N\cdot s}{m^2}$

$\rho = 0.86\left(998\ \frac{kg}{m^3}\right) = 858.28\ \frac{kg}{m^3}$

$\nu = \frac{\mu}{\rho} = \frac{4\times10^{-3}\ N\cdot s/m^2}{858.28\ kg/m^3} = 4.66\times10^{-6}\ \frac{m^2}{s}$

for laminar flow: $Re < 2300$

pipe diameter: $d = 48\text{ in} = 1.22\text{ m}$

pipe cross-sectional area: $A = \pi\left(\frac{d}{2}\right)^2 = \pi\left(\frac{1.22\text{ m}}{2}\right)^2 = 1.169\text{ m}^2$

$Re = \frac{Vd}{\nu} \rightarrow V = \frac{Re\,\nu}{d}$

$V < \frac{(2300)(4.66\times10^{-6}\ m^2/s)}{1.22\text{ m}}$

$V < 8.785\times10^{-3}\ \frac{m}{s}$ for laminar flow.

flow $Q = AV$

$Q = (1.169\text{ m}^2)(8.785\times10^{-3}\ \frac{m}{s})$

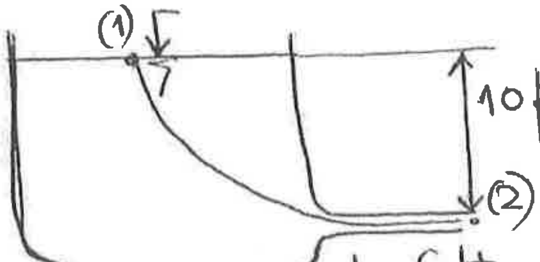
$Q = 1.027\times10^{-2}\ \frac{m^3}{s}$

convert flow Q to barrels/day:

$Q = 1.027\times10^{-2}\ \frac{m^3}{s}\left(\frac{264.17\text{ gal}}{1\text{ m}^3}\right)\left(\frac{1\text{ barrel}}{42\text{ gal}}\right)\left(\frac{86400\text{ s}}{1\text{ day}}\right) = \boxed{Q = 5581.1\ \frac{\text{barrels}}{\text{day}}}$

$\approx 5600\ \frac{\text{barrels}}{\text{day}}$

P4 Problem P6.19



$L = 6\ ft$

$D = \frac{1}{2}\ in \Rightarrow radius = \frac{1}{4}\ in$

$$Q = 35\ ft^3/h$$

$$V = \frac{Q}{A} = \frac{35\ ft^3/\cancel{h}\ \frac{1\cancel{h}}{60\ \cancel{min}}\ \frac{1\ min}{60\ sec}}{\pi\,(0.25\ in)^2 \left(\frac{1\ ft}{12\ in}\right)^2}$$

$$= 7.13\ ft/s$$

$$SG = \frac{\rho_{liquid}}{\rho_{water}} = 0.9 \Rightarrow \rho_{oil} = 0.9 \times 1.94\ \frac{slug}{ft^3} =$$

$$1.746\ \frac{slug}{ft^3}$$

Posing Bernoulli equation between (1) and (2) and disregarding D/2:

$$\underbrace{\cancel{\frac{P_1}{\gamma}}}_{p_{atm}} + Z_1 + \underbrace{\cancel{\frac{V_1^2}{2g}}}_{\approx 0} = \underbrace{\cancel{\frac{P_2}{\gamma}}}_{p_{atm}} + Z_2 + \frac{V_2^2}{2g} + h_f$$

We consider large reservoir $\Rightarrow$

$$h_f = Z_1 - Z_2 - \frac{V_2^2}{2g}$$

$$= 10\ ft - \frac{(7.13)^2\ ft^2/s^2}{2 \times 32.2\ ft/s^2} = 9.21\ ft$$

Now, we assume laminar flow $\Rightarrow$

$$h_f = f\,\frac{L}{D}\,\frac{V^2}{2g} = \frac{64}{Re}\,\frac{L}{D}\,\frac{V^2}{2g}$$

$$h_f = \frac{\cancel{64}^{32}\,\nu}{\cancel{V}D}\,\frac{L}{D}\,\frac{V^{\cancel{2}}}{\cancel{2}_{1}\,g}$$

$$\nu = \frac{h_f D^2 g}{32 L V} = \frac{9.21\ ft\left[0.5\ in\ \frac{1\ ft}{12\ in}\right]^2 32.2\ ft/s^2}{32 \times 6\ ft \times 7.13\ ft/s}$$

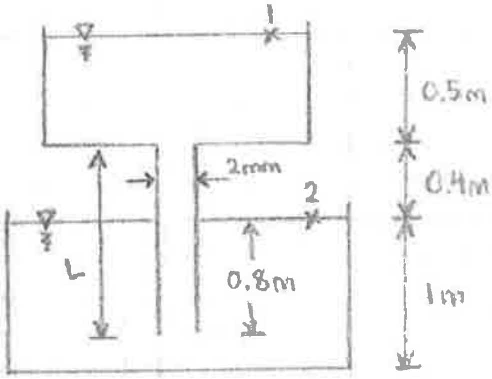
$$= \boxed{3.76 \times 10^{-4}\ ft^2/s}$$

CHECK:

$$\Rightarrow Re = \frac{7.13\ ft/s\ \ 0.5\ in\ \ 1\ ft/12\ in}{3.76 \times 10^{-4}\ ft^2/s} \approx 790 < 2{,}300$$

$\Rightarrow$ the flow is laminar and the assumption was correct

Problem 5 . 6.25. For the configuration shown, the fluid is ethyl-alcohol at 20°C and the tanks are very wide. Find the flow rate which occurs in m^3/hr. Is flow laminar?



ethyl-alcohol at 20°C (Table A.3, White 7th ed.)

$\rho = 789 \ \frac{kg}{m^3}$

$\mu = 0.0012 \ \frac{kg}{m \cdot s}$

Use Bernoulli's from point 1 to point 2

$$\underbrace{\cancel{\frac{P_1}{\rho g}}}_{\text{atmospheric pressure}} + \underbrace{\cancel{\frac{V_1^2}{2g}}}_{\approx 0 \text{ b/c of large reservoir}} + z_1 = \underbrace{\cancel{\frac{P_2}{\rho g}}}_{\text{atm. pressure}} + \underbrace{\cancel{\frac{V_2^2}{2g}}}_{\approx 0 \text{ b/c of large reservoir}} + z_2 + h_f$$

$P_1 = P_2 = P_{atm}$

$V_1 \approx V_2 \approx 0$

$h_f = z_1 - z_2 = 1.9m - 1.0m = 0.9 m$

Now, because pipe diameter d is small, laminar flow is assumed.

Darcy friction factor $f = \frac{64}{Re}$ where $Re = \frac{\rho V d}{\mu}$

From Darcy-Weisbach:

$$h_f = f \frac{L}{d} \frac{V^2}{2g} \Rightarrow h_f = \frac{\overset{32}{\cancel{64}} \mu}{\rho \cancel{V} d} \cdot \frac{L}{d} \cdot \frac{V^{\cancel{2}}}{\cancel{2} g}$$

$$\Rightarrow V = \frac{\rho d^2 g h_f}{32 \mu L}$$

$$V = \frac{(789 \frac{kg}{m^3})(2 \times 10^{-3} m)^2 (9.81 \frac{m}{s^2})(0.9 m)}{32 (0.0012 \frac{kg}{m \cdot s})(1.2 m)}$$

$$V = 0.604 \ \frac{m}{s}$$

Check laminar assumption:

$$Re = \frac{\rho V d}{\mu} = \frac{789 \, kg/m^3 (0.6047 \, m/s)(2 \times 10^{-3} m)}{0.0012 \, kg/m \cdot s} \Rightarrow Re = 795 < 2300 \checkmark$$

Flow rate in $\frac{m^3}{hr}$: $Q = VA = V \cdot \pi \left(\frac{d}{2}\right)^2 = 0.6047 \frac{m}{s} \cdot \pi \left(\frac{2 \times 10^{-3} m}{2}\right)^2 \cdot \frac{3600 s}{1 hr}$

$$\boxed{Q = 6.84 \times 10^{-3} \ m^3/hr}$$

#1

Problem 6

From 6.25

$\rho = 789 \ kg/m^3$
$\mu = 0.0012 \ kg/(ms)$

$L = 1.2 \ m$

Applying Bernoulli:

$$h_f = z_1 - z_2 = 0.9 \ m$$

$$Re_c = 2,300$$

$$Re_c = \frac{VD}{\nu} \implies V = \frac{Re_c \ \nu}{D}$$

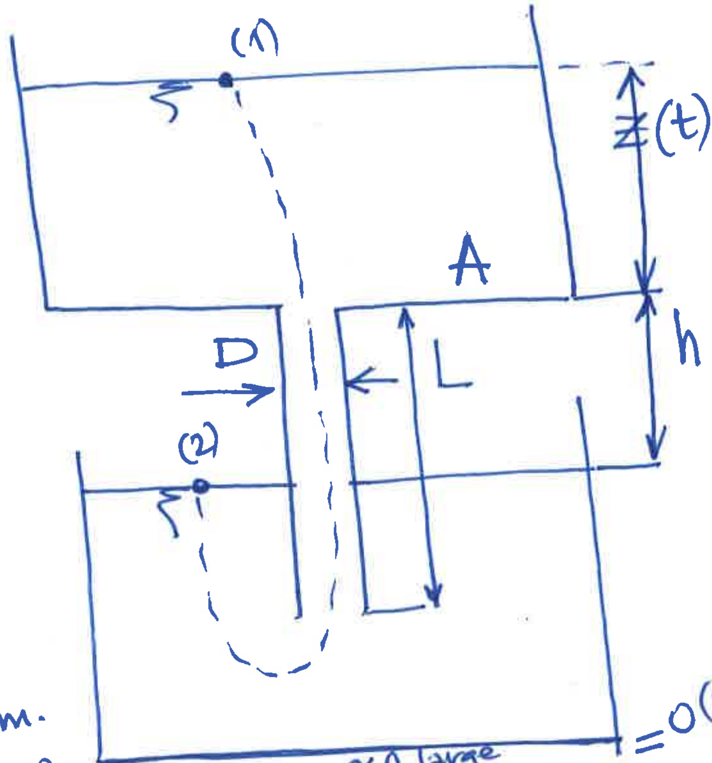
In turn: $h_f = f \frac{L}{D} \frac{V^2}{2g} = f \frac{L}{D} \frac{Re_c^2 \ \nu^2}{D^2 2g}$

But, in laminar flow: $f = \frac{64}{Re_c}$

$$\implies h_f = \frac{64}{\cancel{Re_c}} \frac{L}{D^3} \frac{Re_c^{\cancel{2}} \nu^2}{2g}$$

$$\implies D = \left[\frac{64}{h_f} \frac{L}{2g} Re_c \ \nu^2 \right]^{1/3}$$

$$\boxed{D = \left[\frac{64}{0.9} \frac{1.2}{2 \times 9.81} \ 2,300 \left(\frac{0.0012}{789} \right)^2 \right]^{1/3} = 2.85 \ mm}$$



Problem E1

$$\overset{\text{Patm.}=0}{\cancel{\frac{p_1}{\rho g}}} + z_1 + \overset{\approx 0 \text{ large res.}}{\cancel{\frac{V_1^2}{2g}}} = \overset{=0\,(\text{patm})}{\cancel{\frac{p_2}{\rho g}}} + z_2 + \overset{\approx 0\,(\text{large res})}{\cancel{\frac{V_2^2}{2g}}} + h_{f_{1\to 2}}$$

$$h_{f_{1\to 2}} = \Delta z = z(t) + h \qquad f = \frac{64}{Re} \ (\text{laminar})$$

$$h_{f_{1\to 2}} = f\,\frac{L}{D}\,\frac{V^2}{2g} = \frac{\overset{32}{\cancel{64}}\,\nu}{\cancel{V} D}\,\frac{L}{D}\,\frac{V^{\cancel{2}}}{\cancel{2}g}$$

$$= \frac{32\,\nu\,L\,V}{D^2 g} = z(t) + h$$

$$Q(t) = -A\,\frac{dz(t)}{dt} = -A\,\frac{d}{dt}\left[\frac{32\,\nu L V}{D^2 g} - h\right]$$

$$= -A\,\frac{32\,\nu L}{g D^2}\,\frac{dV(t)}{dt}$$

$$\frac{\pi D^2}{4}\,V(t) = A\,\frac{dz(t)}{dt} \qquad \text{continuity}$$

$$\frac{\pi D^2}{4} V(t) = -A \frac{32 \nu L}{g D^2} \frac{dV(t)}{dt}$$

$$-\int_0^{t^*} \frac{\pi g D^4 \, dt}{128 \nu L A} = \int_{V_0}^{V^*} \frac{dV(t)}{V(t)}$$

$$-\frac{\pi g D^4}{128 \nu L A} t^* = \ln V(t) \Big|_{V_0}^{V^*} = \ln\left(\frac{V^*}{V_0}\right)$$

$$\Longrightarrow \boxed{V(t) = V_0 \left[e^{-\left(\frac{\pi g D^4}{128 \nu L A}\right) t} \right]}$$

Problem E2

$$\frac{p_1}{\rho g} + z_1 + \cancel{\frac{V_1^2}{2g}}^{\approx 0 \text{ large res.}} = \frac{p_2}{\rho g} + z_2 + \frac{V_2^2}{2g} + h_{f1\to 2}$$

$d = 5\ cm$

$$z_2 - z_1 = \frac{0.05}{2} m + 80\ m - 10\ m$$

$$= 70.025\ m$$

$$Q = 60\ \frac{m^3}{h} = \frac{60\ m^3}{3600\ s} = 0.0167\ m^3/s$$

$$V_2 = \frac{0.0167\ m^3/s\ 4}{\pi (0.05)^2 m^2} = 8.488\ m/s$$

$$h_{f1\to 2} = f\frac{L}{D}\frac{V_2^2}{2g}$$

$$Re = \frac{V_2 D}{\nu} = \frac{8.488\ m/s \times 0.05\ m}{10^{-6}\ m^2/s} = 424400$$

For a smooth pipe:

$$f = 0.0132$$

$$h_{f1\to 2} = 0.0132 \times \frac{170\ m}{0.05\ m}\frac{(8.488)^2\ m^2/s^2}{2\times 9.81\ m/s^2} = 164.8\ m$$

$$\frac{p_1}{\rho g} = 70.025\ m + \frac{8.488^2}{2\times 9.81} m + 164.8\ m$$

$$+ \frac{101{,}300}{1000\times 9.81} m = 248.82\ m$$

$$\Longrightarrow \boxed{p_1 = 248.82\ m \times 10^3 \frac{kg}{m} \times 9.81\ m/s^2 = 2.44\ MPa}$$